A MATHEMATICAL MODEL OF OPTIMAL TAX INSPECTION

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A game-theoretical model of tax inspection is considered. Stable forms of behavior of the entrepreneurs and the tax inspectors are identified as a function of the strategy adopted by the central tax service. An optimal central strategy is proposed, which maximizes the average tax revenue for given entrepreneur incomes.

INTRODUCTION

The creation of an efficient functioning tax system is one of the most topical problems during the transition from planned to market economies. Normal operation of social services, health care, education, science, culture, and government organs is closely linked with the existence of a carefully designed system of taxation and tax collection. Unfortunately, no such system exists in Russia today. The established popular opinion that it is both useful and necessary to cheat the government also plays a negative role in this context. Tax evasion is therefore quite common, causing severe losses to the state budget.

Problems with tax collection are not restricted to Russia. Despite the long tradition of taxation in the Western countries, the existing tax systems are far from optimal, and government budgets suffer considerable losses through tax collection shortfalls (see [1, 2] and elsewhere).

The creation of an efficient tax system requires the solution of two interconnected problems. The first is the determination of the structure of taxes and the tax rates. These parameters must be specified based on the overall conception of socio-economic development, including the principles of distribution of national income between the market and the budget spheres of the economy. Redistribution of national income through taxation has been considered by many authors (see [3, 4] and elsewhere). Our paper does not touch on this problem, assuming in what follows that the total income to be retained in the market sphere is given. We only note that in our view the interests of the great majority of Russian population require a rapid and substantial increase of financing allocated to the budget activities listed above.

The second problem involves optimal organization of tax inspection. The variable to be optimized is the total budget revenue from tax collection less the cost of operating the tax inspection and subject to maintaining a given level of entrepreneur incomes. In the process of organization of tax inspection we have to determine various parameters, such as the penalties for tax evasion, the frequency and the procedure for auditing the tax returns, internal audit procedures, the number of inspectors, inspector salaries, etc.

The present paper constructs and analyzes a model of tax-collection audit. It examines the interaction between entrepreneurs, tax inspectors, and the central tax service monitoring the inspectors, and conducts a game-theoretical analysis of this interaction in order to determine the optimal values of the relevant parameters. Sec. 1 describes the model, Sec. 2 presents the necessary information from game theory, Sec. 3 analyzes the stable forms of behavior of the players, and Sec. 4 compares the different forms of behavior in terms of the revenues collected by the central tax service. Substantive conclusions regarding reorganization of tax inspection are presented in Sec. 5.

Note that the proposed model can be applied to study the qualitative behavior of the players, but cannot be used to produce exact values of the control parameters, because it represents the interaction in a highly simplified form. In particular, it ignores the nonhomogeneity of the group of entrepreneurs and the group of inspectors. Yet the results of our study are also relevant for the organization of other inspection services, such as customs, law enforcement, etc.
1. THE MODEL

1.1. The Interaction Scheme and the Main Model Parameters

The proposed model of tax inspection describes the following interaction between entrepreneurs, tax inspectors, and the central tax service ("the center").

In the first stage, the entrepreneurs supply to the center information about their income. It is assumed that the income of each entrepreneur is a random variable that takes the values $V$ or $V + V'$ with known probabilities. If $M$ is the total number of entrepreneurs and $M$ is the average number of enterprises with income $V + V'$, then the probability of earning $V + V'$ is $X = \frac{M}{M}$ and the probability of earning $V$ is correspondingly $1 - X$. The income $V$ is not taxed, while each entrepreneur earning the income $V + V'$ is required to pay the amount $nV'$ in taxes. Having earned the income $V$, the entrepreneur reports this fact to the center and is exempted from tax payment. Having earned the income $V + V'$, the entrepreneur may declare it in full or may evade the tax by not reporting the incremental amount $V'$ in his tax return to the center.

In the second stage, the inspectors audit a number of entrepreneurs assigned to them for the current period and report their income to the center. The audited entrepreneurs are sampled at random. The specific form of reporting by the inspector can be described as follows. Depending on the income earned by the audited entrepreneur and the original tax return, the inspector either directly reports the entrepreneur's true income to the center or suggests to reach an agreement, asking for a bribe in order to confirm the original tax return. Thus, when an inspector discovers an instance of tax evasion, he may suggest to "close his eyes" in return for a bribe. If the true income is only $V$, the inspector may still attempt to extort the amount $\bar{D}$ from the entrepreneur by threatening to report a higher income $V + V'$ to the center. At this stage, the inspector decides whether to make an offer of an agreement and determines the amount of the bribe.

In the third stage of the interaction, the entrepreneur responds to the inspector's offer, either agreeing or not agreeing to pay the bribe. The situation when the inspector agrees to conceal the attempted tax evasion is called bribery in what follows; the situation when the inspector demands a bribe in order to confirm a true low income $V$ is called extortion. If the entrepreneur refuses to reach an agreement, we assume that in the first instance the inspector reports the true income to the center and in the second instance the inspector assigns the income $V + V'$ to the entrepreneur.

Finally, in the fourth stage of the interaction the center audits a certain number of inspectors and based on the results of this audit collects the penalties for the detected violations. For the inspector these violations are bribery and extortion, and for the entrepreneur the violation is attempted tax evasion. The information reported by entrepreneurs who are not audited by inspectors and by inspectors who are not audited by the center is assumed true. The center cannot be bribed, and the audits conducted by the center are completely reliable.

The central strategy thus includes selection of the following parameters:

$\rho$ - the probability of auditing one entrepreneur during the current period; this probability uniquely determines the required number of tax inspectors $K = (\bar{M}\rho/m) + 1$;

$\rho_c$ - the conditional probability of a repeat audit by the center;

$n$ - the tax rate or the proportion of excess income collected into the budget;

$P$ - the penalty for an entrepreneur found guilty of tax evasion;

$S$ - the inspector's salary;

$T$ and $\bar{T}$ - penalties for inspectors found guilty of bribery and extortion, respectively.

Denote by $L$ the cost of one audit conducted by the center. The exogenous parameters of the model also include the admissible minimum value of the average net income of the entrepreneurs and the inspectors, $\theta$ and $S_{\text{min}}$ respectively. As noted in the Introduction, $\theta$ is established as part of the general conception of socio-economic development. The determination of $S_{\text{min}}$ in practice is based on the alternative income that inspectors can earn in other jobs.

The income of the players as a result of the interaction is a random variable that depends on their behavior strategy. Thus, the entrepreneur income is the original income less taxes, payments to the inspector, and penalties; the inspector income is the salary plus possible payments from the entrepreneurs less any penalties. The central income is equal to tax collection plus penalties less the inspector salaries and the costs of monitoring the inspectors.

The objectives of our study are 1) to analyze the stable forms of behavior of entrepreneurs and inspectors; 2) to determine the optimal strategy of the center that maximizes the revenue subject to the condition that the average income of entrepreneurs and inspectors is not less than $\theta$ and $S_{\text{min}}$ respectively. The central strategy is assumed known to all players.