Hierarchical interaction in design systems

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Multilevel hierarchical schemes for complex system design are considered. Coordination of engineering-design decisions made at different levels is examined. An interactive design-seeking procedure is described.

1. Statement of the Problem

Given is a finite set $X \subseteq \mathbb{E}^N$, which describes the set of alternative technical systems with $N$-dimensional vectors of design parameters $x \in X$. The comparative efficiency of the alternatives is determined by the binary relation $\Phi$ on $X$.

The design problem is to select one maximum alternative $x_*$ from $X$ in the choice model $(X, \Phi)$ [1]:

$$x_* \in X_* = \max(X, \Phi).$$

We assume that $X_* \neq \emptyset$. Problem (1.1) is a general case of optimal design problems [2], including multicriterion design problems.

Direct solution by computer of problem (1.1) for complex systems is ruled out because of the high dimension $N$ of the vector of design parameters $x$ and the complex specification of $X$. To overcome the difficulty, we construct a hierarchy of design problems. We introduce new design parameters $z^{l+1} = f_{l+1}(z^l)$, $X^{l+1} = f_{l+1}(X^l)$, where $z^l \in X^l \subseteq \mathbb{E}^{N_l}$, $N_{l+1} \leq N_l$, $l = 0, s - 1$. Each vector $x^{l+1}$ is obtained from $x^l$ by aggregation using the vector function $f_{l+1}$ and it gives a more holistic description of the system being designed than $x^l$. Here $s$ is the number of aggregation levels.

The binary relation $\Phi$ on $X$ is defined by the vector efficiency criterion $W(z) = (W_1(z), \ldots, W_n(z)) : z_0 \preceq y \Leftrightarrow W_i(z) > W_i(y)$, $i = 1, m$ (Slater relation) [1]. To each level $l$ we associate an $m_l$-dimensional vector criterion $W_l(x^l)$ which defines the corresponding Slater relation $\Phi_l$ on $X^l$. Problem (1.1) is solved by the following decomposition scheme:

$$X^l_* = \max(f_{l+1}^{-1}(X_*^{l+1}), \Phi_l), \quad l = s - 1, \ldots, 0, \quad W_0 = W,$$

with the initial condition

$$X^s_* = \max(X^s, \Phi_s).$$

Any element of the set $X^s_*$ is accepted as a solution. It is shown in [1] that under certain natural conditions this procedure produces a solution of the original design problem (1.1).

The hierarchical decomposition approach proposed above requires elaboration. Even with scheme (1.2) the number of elements of the set $X^l_*$ may be too large for implementation in CAD systems, and subsets of the corresponding sets (possibly one-element subsets) may have to be used. Based on this remark, we obtain the scheme [3, 4]

$$X^l_j \rightarrow z^l_j \in X^l_j, \quad z^l_j \rightarrow X^{l-1}_j = \max(f_j^{-1}(z^l_j), \Phi_{j-1}), \quad j = s, \ldots, 0,$$

where $X^s = \max(X^s, \Phi_s)$.

Uncoordinated choice of alternatives $x^j_*$ obviously will not solve the design problem (1.1), and interactive control is therefore included in the scheme (1.3) in [3, 4]. The scheme proposed in [4] controls the process by choosing the hierarchy level $l$ ($l = 0, \ldots, s$) and the vector $\mu^l$, called the system "conception" vector. The choice of the vector $\mu^l$ that folds the particular criteria $W_l(\cdot)$ into a general criterion.
\[ U^l(\mu^l, \cdot) = \min_{1 \leq i \leq m_1} \mu^l_i \cdot W^l_i(\cdot), \] (1.4)

is interpreted as formation of a system conception at level \( l \) of the hierarchy. This has the following interpretation.

Assume that each criterion \( W^l_i(\cdot), i = 1, m_l \), estimates the suitability of the system to perform each of its tasks and/or a certain required quality of the system. For instance, in aircraft design, \( W^l_i(\cdot) \) may represent the probabilities of various flight assignments.

Optimization of the system design parameters by the criterion (1.4) determines a combination of system qualities with "weights" \( \mu^l_i \).

In [3], control involves choosing some set \( I_l \) of indexes \( i \) from the set \( I_l = \{1, \ldots, m_l\} \cup \{\emptyset\} \), for which the decision maker attempts to increase the value of the particular criteria \( W^l_i \). These controls are chosen from special considerations (see Sec. 2), which are called "design laws" in [3].

The interaction methods proposed in [3, 4] for controlling the design process involve review of the project on less detailed levels of the description hierarchy. Given methods for reviewing design decisions and constructing the system "conception", we can develop a procedure that combines the approaches of [3] and [4].

2. General Description of Interactive Procedure

The interactive decision procedure incorporated in a hierarchical CAD system implementing the decision process (1.3) consists of three main blocks. We briefly describe each procedure block.

**Procedure 0**

**0.0. Initialization.** Set \( j = 0, L_0 = \{0, \ldots, s\}, \Omega^0 = M^0, H^l = H_0, \ l = 0, s, L := L_0 \), where \( j \) is the iteration counter, \( M_0 = \{\mu^0 \in E^m, \mu^0 > 0, i = 1, m_0, \sum_{i=1}^{m_0} \mu^0_i = 1\} \), \( W^0 = W \), \( H_0 \leq E^m \).

**0.1.** Set \( j = j + 1, \) take \( l \in L \). If \( l \in L \) (\( l \) is the set of as yet unconstructed levels of the hierarchy generated by procedure 1), then \( h^l := 0 \) and go to 0.2, else go to 0.6.

**0.2 (generation of the \( l \)-th level set of alternatives).** If \( l = s \), then \( X^s \subseteq X^s \) and go to 0.3.

Let \( 0 \leq l < s \). If \( x^{l+1} \) has been determined, then set \( X^l := f_{l+1}^1(x^{l+1}) \) and go to 0.3; else the set of alternatives is undefined and go to 0.1.

**0.3 (construction of the \( l \)-th level system).** Transfer control to procedure 1. If procedure 1 finds an alternative \( x^l \) that satisfies the designer's requirements, then \( h^l := h^l + 1, \) \( X_{l+1}^l := X^l \) and go to 0.4. (The variable \( h^l \) is the number of sets \( X^l \), where \( X^l = \text{Max}(f_{l+1}^1(x^{l+1})), W^l \) on the \( l \)-th hierarchical level, that arise in the process of examination of various search scenarios.) Otherwise, \( x^l \) has not been determined because of improper actions by the decision maker. Then set \( l := l + 1, j := j + 1 \) and go to 0.6.

**0.4 (exclusion of a priori "bad" alternatives).** If \( h^l = 1 \), then go to 0.5; else set \( X^l_i := X^l \setminus z^l_i \), where \( z^l_i \in X^l \), such that there exists \( z^l_k \in X^l, \) for which \( W^l_k(z^l_k) > W^l_k(z^l_i) \), where \( 1 \leq i, k \leq h^l, k \neq i, p = 1, \ldots, m_l \).

**0.5 (stopping rule).** If \( l \neq 0 \), then \( l := l - 1, j := j + 1 \) and go to 0.2; else exit the procedure, \( x^l \) is a solution of the problem.

**0.6.** This step of the interactive procedure is reached when level \( l \) has been chosen for analysis and some previous iteration has produced the set \( X^l \), where \( i = 1, \ldots, h^l \). If the decision maker starting from this level wishes to examine a different search scenario, choosing an alternative \( x^l \) which is different from previously examined alternatives, then we have to compute

\[ \mu^l_i = [W^l_i \sum_{i=1}^{m_l} (W^l_i)_{-1}]^{-1}, \ i = 1, \ldots, m_l, \]

\[ \Omega^l = \Omega^{l-1} \cap \omega^l_i(\mu^l_i), \]

where \( W^l_i = W^l(x^l_i) \), the mapping \( \omega^l_i \) is determined in procedure 1. If \( \Omega^l \neq \emptyset \), then \( j := j + 1 \) and go to 0.2; else the choice is bad and go to 0.6. Otherwise the decision maker prepares to increase the value of \( W^l_i(x^l_i) \) for any \( i \in I \subseteq I^l = \{1, m_l^l\} \cup \{\emptyset\} \).