The paper deals with questions of bounds on mixed semi-invariants of a random process $x_t$ which satisfies a mixing condition either "according to Rosenblatt" or "according to Ibragimov."

For many problems related to the limiting behavior of sums of random variables it is important to know the bounds on the mixed semi-invariants of random process $x_t$. In [1-4] the questions were investigated dealing with the bounds of mixed semi-invariants depending on the maximal distance between the temporal coordinates of the semi-invariant. Such bounds are obtained for Markov chains as well as for random processes subject to mixing conditions of the almost Markovian type. An example was adduced in [4] showing that, under mixing conditions either "according to Rosenblatt" or "according to Ibragimov," it is impossible to obtain the aforementioned semi-invariant bounds. In the present paper we investigate questions of bounds of mixed semi-invariants of random process $x_t$ subject to one of the following mixing conditions:

1. Mixing conditions of process $x_t$ "according to Rosenblatt"

$$
\tau (\tau) = \sup_{\Lambda \in \mathcal{B}_t} |P(AB) - P(A)P(B)|_{1,\infty} \to 0,
$$

(1)

2. Mixing condition of process $x_t$ "according to Ibragimov"

$$
\beta (\tau) = \sup_{\Lambda \in \mathcal{B}_t} |P(A|B) - P(A)|_{1,\infty} \to 0,
$$

(2)

where we have denoted by $\mathcal{B}_t$ the $\sigma$-algebra generated by the random variables $x_t$, $t \in [a, b]$.

Consider random processes $x_t$, $t \in \mathbb{R}$, where $\mathbb{R}$ is the set of real numbers. Let $\mathcal{F}_t$ be the $\sigma$-algebra generated by the random variables $x_t$, $t \in [s, t]$. We denote by $C^\omega$ the space of complex functions which are nonzero on a finite set of points. We denote by $N^\omega$ the space of nonnegative integer-valued functions which are nonzero on a finite set of points. We set, for $\varphi \in C^\omega$ and $\psi \in N^\omega$,

$$
\varphi^\omega = \prod \varphi (t)^{\psi (t)}, \text{ considering } \varphi^0 = 1;
$$

$$
\psi! = \prod \psi (t)!, \text{ considering } \psi! = 1
$$

and

$$
|\psi| = \sum |\psi (t)|, \varphi, \psi \in C^\omega (\varphi \psi) = \sum \varphi (t) \psi (t).
$$

It follows from the two formal expansions

$$
f (\lambda) = M (x^{\omega}) = 1 + \sum_{\lambda \in \mathbb{N}^\omega} M_\lambda \frac{\lambda^{\omega}}{\lambda!}, \text{where } M_\lambda = M (x_{\lambda}),
$$

$$
g (\lambda) = \ln f (\lambda) = \sum_{\lambda \in \mathbb{N}^\omega} A_\lambda \lambda^{\omega}
$$

that the coefficients \( A_\mu \) and \( M_\alpha \) are connected by the formula
\[
A_\mu = \sum_{\alpha \in N^\omega} \frac{(-1)^{i-1}}{i} \frac{M_{x_i} \cdots M_{x_1}}{\alpha_1 \cdots \alpha_i},
\]
which turns out to be true if only all the moments entering on the right of Eq. (3) exist, the formula being a consequence of the following formal transformations:
\[
g(\lambda) = \ln f(\lambda) = \ln (1 + f(\lambda) - 1) = \sum_{i=1}^{\infty} \frac{(-1)^{i-1}}{i} (f(\lambda) - 1) = \sum_{i=1}^{\infty} \frac{(-1)^{i-1}}{i}.
\]

The following is valid.

**Lemma 1.** Let \( g(t) = 0 \) for \( t = t_1, \ldots, t_l \). If the collections of random variables \( x(t_1), \ldots, x(t_l) \) are independent, then \( A_\mu = 0 \).

**Proof.** Using the independence of the variables \( x(t_1), \ldots, x(t_l) \) and \( x(t_{l+1}), \ldots, x(t_{l+\nu}) \), we obtain
\[
g(\lambda) = \ln M_{x(\omega)} = \ln M_{x(T_{t_l}^{(t_{l+1})} x(t_{l+1}) x(t_{l+\nu}))} = \ln M_{x(T_{t_l}^{(t_{l+1})} x(t_{l+1}))} + \ln M_{x(T_{t_l}^{(t_{l+1})} x(t_{l+\nu}))}.
\]

The coefficients of the \( \lambda^\mu \) in the expansion of \( g(\lambda) \) are found from the relationship
\[
A_\mu = \frac{1}{\mu!} \frac{\partial^{\mu} g(\lambda)}{\partial \lambda^\mu},
\]
the right side of which, obviously, equals zero if \( g(\lambda) \) can be presented in the form of (5). Lemma 1 is proven.

**Lemma 2.** If the following equation is true for all \( \nu \in N^\omega \),
\[
M_{x(\nu)} = M_{x_{T_{t_l}^{(t_{l+\nu})}}} M_{x_{T_{t_l}^{(t_{l+\nu})}}},
\]
then \( A_\mu = 0 \).

**Proof.** We introduce the new process \( \tilde{x}_t \), all finite-dimensional distributions of which can be presented in the form
\[
P(x_{t_1} \in A_1, \ldots, x_{t_l} \in A_l) = P(x_{t_1} \in A_1, \ldots, x_{t_m} \in A_m) P(x_{t_{m+1}} \in A_{m+1}, \ldots, x_{t_l} \in A_l),
\]
where \( t_1 < \ldots < t_m < t_{m+1} < \ldots < t_l \). Then, \( M_{x(\nu)} = M_{x_{T_{t_l}^{(t_{l+\nu})}}} \), which follows from formula (6). Consequently, \( A_\mu(\tilde{x}_t) = A_\mu(x_t) \). It follows from Lemma 1 that \( A_\mu(\tilde{x}_t) = 0 \). Then, \( A_\mu(x_t) = 0 \), Q.E.D.

Let \( \mu(t) \neq 0 \) for \( t = t_1, \ldots, t_l \).

**Theorem 1.** If, for some \( k > 2 \),
\[
M |x^{(\nu)}_{t_i}| \leq C,
\]
then the following inequality is valid:
\[
|A_\mu| \leq \frac{\mu^{|\nu|+1}}{\mu!} [z (\max_i |t_{i+1} - t_i|)^{1-\nu} C^\nu (4 + 6C)].
\]