In a general stochastic model of epidemics it is assumed that the initial number of patients is finite, and that with increasing size of the population the control parameter approaches a constant. Under these conditions we study the properties of the limiting distribution of the size of an epidemic.

1. Statement of Problem and Results. Suppose that at the initial instant of time a population contains $n$ persons who are susceptible to infection and $m$ sick persons. We shall assume that the evolution of the epidemic in time is a Markov process with continuous time and states of the form $(r, s)$, where $r$ is the number of susceptible and $s$ the number of sick persons. We shall also assume that a transition from a state $(r, s)$ to a state $(r - 1, s + 1)$ during an infinitely small time interval $dt$ takes place with a probability $\lambda rs \, dt + o(dt)$, whereas a transition from a state $(r, s)$ to a state $(r, s - 1)$ takes place with a probability $\mu s \, dt + o(dt)$. It is easy to see that a transition of the first type signifies that yet another person among the susceptible ones has become sick, whereas a transition of the second type signifies the removal of a sick person from the population (due to curing with immunity, isolation, or death).

The ratio $\rho = \frac{\mu}{\lambda}$ is called the relative removal coefficient.

Such models of epidemic have been studied by many investigators (see, for example, [1]).

The above process terminates when a state of the form $(k, 0)$ is reached. It is evident that under our assumptions this will occur sooner or later. The size of the epidemic $\nu^m_n$ is defined as the number of persons (out of the initial number of susceptible ones) who get sick prior to the termination of the epidemic.

The aim of this note is to study the properties of $\nu^m_n$ on the assumption that

$$n \to \infty, \quad \rho/n \to 0 \to \text{const},$$

and in contrast to [2] we have

$$1 \leq m \leq m_0 < \infty.$$

The assumptions (1) and (2) retain their validity throughout this paper.

It is natural to say that an epidemic degenerates with a probability $P$ if for any finite $k$ there exists

$$\lim_{n \to \infty} P\left(\nu^m_n = k\right) = p^m_k$$

with

$$\sum_{k=0}^{\infty} p^m_k = P^m = P.$$

$P = P^m$ will be called the probability of degeneration of the epidemic. We have the following

THEOREM 1. Under our conditions

$$P^m = \min \left(1, \theta^m\right)$$

and

\[ p_k^{(m)} = \frac{m}{2k+m} \left( \frac{\theta}{1+\theta} \right)^{k+m} \left( \frac{1}{1+\theta} \right)^{m}. \]

For the case \( m = 1 \), this theorem was obtained by T. Williams [3].

It follows from condition (1) and Theorem 1 that the parameter \( \rho/n \) is the control parameter of the epidemic. If \( \rho/n \geq 1 \), the epidemic degenerates with probability 1, and if \( \rho/n < 1 \) (in the sense that \( \theta < 1 \)), then the epidemic breaks out with a probability \( 1 - P^{(m)} > 0 \). The strong similarity to branching processes is not accidental. This is due to the fact that during a fairly long period of its development, the epidemic process under consideration is almost indistinguishable from a linear birth-and-death process. As is well-known, the latter is a particular case of a branching process.

Let \( \alpha \) be the smaller positive root of the equation

\[ \alpha + (\rho/n) \ln (1 - \alpha) = 0, \quad (5) \]

and \( \alpha_0 \) the smaller positive root of the equation

\[ \alpha + \theta \ln (1 - \alpha) = 0. \quad (5') \]

Let us write

\[ \rho = \alpha_0 \bigg( 1 + \frac{p}{(1 - \alpha_0)} (\theta/(1 - \alpha_0) - 1)^{-1} \bigg). \quad (6) \]

The next theorem describes the limiting properties of the size of an epidemic if the latter does not degenerate.

**THEOREM 2.** If \( \theta < 1 \), then

\[ P \{ v_k^{(m)} < n\alpha + x_1 \sqrt{n} \} = P^{(m)} + (1 - P^{(m)}) \Phi(x) + o(1) \]

uniformly in \( x \), \( x \approx -(n\alpha - l_n)/\sigma \sqrt{n} \).

Here

\[ l_n = l(n), \]

\( l(n) \) being a slowly increasing function, and \( \Phi(x) \) a standard normal distribution law.

The theorem just formulated has two simple corollaries.

**COROLLARY 1.** Under the conditions of Theorem 2,

\[ P \{ v_k^{(m)} < n\alpha + x_1 \sqrt{n} \} = \Phi(x) + o(1) \]

uniformly in \( x \), \( -\infty < x < \infty \).

Thus the proportion of individuals in the population who got sick prior to the termination of the epidemic is equal to \( \alpha \) with a probability \( 1 - P^{(m)} \), and to zero with a probability \( P^{(m)} \).

**COROLLARY 2.** If in the condition (1) we have

\[ \theta - \rho/n = o(n^{-1}), \quad (1') \]

then, according to Theorem 2, it is possible to replace \( \alpha \) by \( \alpha_0 \).

If an epidemic breaks out, the control parameter \( \rho/n \) can be estimated according to its termination. For this purpose we shall write

\[ \alpha_1 = \frac{\theta + 1 - z_0}{\theta - 1 - z_0} \quad (7) \]

and introduce the random variable

\[ \theta_n^* = \frac{1 - v_n^{(m)/n}}{v_n^{(m)/n}}. \quad (8) \]

We have the following