We consider the problem of calculating the absolute projection constants $\lambda(X^n)$ of the symmetric $n$-dimensional spaces $X^n$. We find an integral representation of the projectors belonging to a class sufficient for calculation of the projection constants. The formula so obtained is used to calculate the absolute projection constant of one of the Marcinkiewicz spaces and, by so doing, to give a negative answer to a question raised by Grünbaum concerning the asymptotic behavior of the quantity $\lambda_n = \sup_{X^n} \lambda(X^n)$.

Let $Z$ be a real Banach space, and let $X = X^n$ be a finite-dimensional subspace of $Z$. Let $\pi$ be the set of bounded linear projectors of $Z$ onto $X$ and let $Z(X)$ be the set of all real Banach spaces containing $X$ as a subspace of $Z \in Z(X)$. The quantity $\lambda(X; Z) = \inf_{P \in \pi} \|P\|$ is called the relative projection constant [1].

By the absolute projection constant we mean the quantity

$$\lambda(x) = \sup_{Z \in Z(X)} \lambda(X; Z).$$

Let $C(Q) \in Z(X)$ be the space of all continuous functions on the metric compactum $Q$. As noted in [1] and proved in [2], $\lambda(X) = \lambda(X; C(Q))$. In [3-5] the following constants were evaluated:

$$\lambda(l_2^n) = \frac{n! \left(\frac{n}{2}\right)}{\sqrt{\pi} \Gamma\left(\frac{n+1}{2}\right)} \sim \sqrt{\frac{2n}{\pi}};$$

$$\lambda(l_2^{2k-1}) = \lambda(l_2^{2k-1}); \quad \lambda(l_2^k) = \lambda(l_2^{2k-1}).$$

We put

$$\lambda_n = \sup_{X^n} \lambda(X^n),$$

where $X^n$ ranges over all $n$-dimensional Banach spaces.

M. I. Kadets and M. G. Sibirskii [1] showed that

$$\lambda_n \lesssim \sqrt{n}.$$  \[1\]

B. Grünbaum [6] raised a question, which, in our notation, can be formulated as follows: is it true that

$$\lambda_n \sim \lambda(l_2^n).$$

We obtain a formula for calculating the absolute projection constants of a class of spaces (symmetric spaces); we calculate the absolute projection constant of a Marcinkiewicz space and therewith give a
negative answer to Grünbaum's question. We fix a basis $e_1, e_2, \ldots, e_n$ in the linear space $X^n$. Let $y \in X^n$, $y = \sum_{i=1}^{n} y_i e_i$. We put

$$y_i = \sum_{j=1}^{n} e_j y_i.$$

where $I = (i_1, i_2, \ldots, i_n)$ is a permutation of the segment $1, 2, \ldots, n$; $e = (e_1, e_2, \ldots, e_n)$, $e_i = \pm 1$.

We call a mapping $T : X^n \rightarrow X^n$ of the form $Ty = y_I$ a symmetric mapping. The set of symmetries produced by the operation of superposition forms a group of $2^n n!$ elements.

We say that $e_1, e_2, \ldots, e_n$ is a symmetric basis and that $X^n$ is a symmetric space [7] if

$$\|Ty\|_{X^n} = \|y\|_{X^n} \text{ for arbitrary } y \in X^n, T \in S.$$

Let $K$ be the closure of the set of boundary points of the unit ball of the conjugate space $(X^n)^*; X^n \subset C(K)$ is a natural imbedding.

As noted above,

$$\lambda(X^n) = \lambda(X^n; C(K)).$$

Let $X^n$ be a symmetric space. Then the set $K$ is invariant relative to symmetries and the group $S$ acts on $C(K)$ according to the rule

$$Tf(y) = f(T^{-1}y), \quad f \in C(K), \quad T \in S.$$ We say the projector $P : C(K) \rightarrow X^n$ is symmetric if $P(Tf) = TPf$ for arbitrary $f \in C(K)$, $T \in S$. We denote the set of all symmetric projectors by $\pi_S$.

**PROPOSITION.** Let $X^n$ be a symmetric space. Then

$$\lambda(X^n; C(K)) = \inf_{P \in \pi_S} \|P\|.$$

**Proof.** Let $P : C(K) \rightarrow X^n$ be an arbitrary projector. Consider the mapping

$$P_S = \frac{1}{2^n n!} \sum_{T \in S} T^{-1}PT.$$

Obviously, $P_S$ maps $C(K)$ into $X^n$ and

$$\|P_S\| \leq \frac{1}{2^n n!} \sum_{T \in S} \|T^{-1}PT\| \leq \frac{1}{2^n n!} \sum_{T \in S} \|PT\| \|T^{-1}\| = \|P\|.$$

We show that $P_S \in \pi_S$. For $y \in X^n$

$$P_S(y) = \frac{1}{2^n n!} \sum_{T \in S} T^{-1}PTy = \frac{1}{2^n n!} \sum_{T \in S} T^{-1}(Ty) = y.$$

For $\bar{T} \in S$

$$P_{S\bar{T}} = \frac{1}{2^n n!} \sum_{T \in S} T^{-1}PT\bar{T} = \frac{1}{2^n n!} \sum_{T \in S} (T\bar{T})^{-1}PT\bar{T} = \bar{T}P_S.$$

This completes the proof of the proposition.

Let $M$ be the set of vector-valued measures $\mu$ with values in $X^n$, defined on the $\sigma$-ring $\mathcal{U}$ of Borel subsets of $K$ and satisfying the condition

$$\mu(TA) = T\mu(A) \text{ for arbitrary } T \in S, A \in \mathcal{U}. \quad (1)$$

**THEOREM.** Let $X^n$ be a symmetric space, and let $P$ be a mapping of $C(K)$ into $X^n$. A necessary and sufficient condition in order that $P \in \pi_S$ is the existence of the representation