THE WORK FOR FAILURE AND THE WORK FOR PLASTIC
DEFORMATION IN FRACTURE-TOUGHNESS TESTS

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Experimental determination of the J integral is finding increasing use in laboratory practice and, therefore, it would be useful to dwell on some of the basic assumptions relative to its rating and interpretation. Frequently the critical value \( J_c \) of this integral is taken as the unit work expended in failure. However, treatment of the characteristics \( J_c \) and \( J_{1c} \) obtained in the case of strong plastic flow of a sample as the unit for failure (similarly to the fracture toughness \( G_c \) and \( G_{1c} \)) causes doubt. Accordingly, it is impossible to calculate reliably and confidently the parameter of fracture toughness \( K_{1c} \) according to the equation

\[
J_{1c} = [(1 - \nu^2)/E] K_{1c}^2.
\]

The reasons for such apprehension is the fact that the J integral has been introduced for the case of the deformation theory of plasticity, which does not allow unloading and which, in essence, is the nonlinear theory of elasticity. The method of experimental determination of the J integral on a series of samples with different crack lengths \( l \) is completely correct since in these tests the J integral is the difference in potential energies \( U \) of two bodies with crack lengths differing little (referred to the difference in areas of the crack): 

\[
J = -(1/t) dU/dl.
\]

However, calculation of the same value on a single sample during growth of a crack, particularly according to the equation (in bend or off-center tensile tests) 

\[
J = \left[2A/(t(b - l_0)) \right] (A \text{ is the area of the } P \text{ vs } A \text{ curve, } P \text{ is force, } A \text{ is displacement of the point of application of the force, } b \text{ is the width of the sample, } t \text{ is its thickness, and } l_0 \text{ is the initial crack length})
\]

is undesirable since with advance of the crack there are formed new surfaces free from load. At the same time, in some volume surrounding the tip of the crack the stresses relax. Unloading occurs in the plastically deformed material according to the rule of linear elasticity, which leads to a breakdown in the basic positions of the deformation theory of plasticity. In addition, with weak advance of the crack the flow of energy to its tip (for accomplishment of the mechanism of failure) is a flow of elastic energy and, consequently, first, it is not equal in a plastically deformed sample to the J integral and, secondly, it is impossible to identify the J integral with the unit work for failure. In failure there is consumed not all of the work of deformation \( A \) (elastic and plastic) but only the elastic. Therefore, for example, in a hard plastic body growth of a brittle crack is impossible.

From this it is clear that to determine the unit energy for failure the method of experimentally finding the J integral must be changed in such a manner that the expenditure of work for plastic deformation not having a relationship to failure (but making a contribution to the J integral) is excluded. Let us consider possible variations of such a separation.

First Variation. As a result of testing a sample with an original crack let us have available two curves, one of them force \( P \) vs displacement \( A \) of the point of application of the force and the other force \( P \) vs crack length \( l \). Also let there be an equation for calculating the elastic pliability \( \lambda = \lambda(l) \) of the given sample (or experimental calibration of \( \lambda \) vs \( l \) curves). Then it is possible to accomplish the following construction. Each point of the \( P \) vs \( l \) curve may be placed in accordance with a value of \( \lambda \). These values of \( \lambda \) on the \( P \) vs \( \Delta \) curve provide a cluster of straight lines originating from the origin of the coordinates (since \( \Delta_0 = \lambda P \) is a linearly elastic displacement) and each of them is related to some crack length \( l_i \). The force corresponding to \( l_i \) may be determined from the \( P \) vs \( l \) curve. Therefore, having plotted the corresponding forces on the lines of this cluster, we obtain a new \( P \) vs \( \Delta \) curve in which the displacement \( \Delta \) is the elastic displacement \( \Delta_\lambda \) (Fig. 1). The area of the cross-hatched curved triangle in Fig. 1 is the elastic energy expended in advance of the crack from \( l_0 \) to \( l_c \). Referring this area to the area of the newly formed surface of the crack \( (l_c - l_0) \), let us find the average unit work for failure \( G_c \) (or \( G_{1c} \) in failure by rupture).

The problem of separating the displacement \( \Delta \) and the work \( A \) into the elastic and plastic components has been solved. We should note that the ratio of the areas \( A_p \) of the \( P \) vs \( \Delta p \) curve and \( A_e \) of the \( P \) vs \( \Delta e \)
curve may serve as a criterion for separation of the state of a material into brittle \((\Delta p/\Delta e < 1)\) quasibrittle \((\Delta p/\Delta e \approx 1)\), and plastic \((\Delta p/\Delta e > 1)\).

Second Variation. It is possible to point out an approximate but similar method of constructing the P vs \(\Delta e\) curve. Let there be only a P vs \(\Delta\) curve recorded on equipment on the basis of movement of the clamps. Recording the curve, we remove the load (with recording of the unloading line) and complete the test at the point \(P = P_C\) and \(l = l_C\) located at an arbitrary point on the descending branch of the curve (Fig. 2). We displace the unloading line obtained so that it is parallel to this same line at the origin of the coordinates. We bring down the force \(P_C\) along the horizontal to the line \(l = l_C\) and the force \(P_{\text{max}}\) to the line \(l = l_0\). The area \(\Delta e\) of the cross-hatched triangle in Fig. 2 provides approximately the sought-for elastic energy. Then \(G_C\) is calculated similarly to the calculation in the first variation.

It must be expected that making more accurate the form of the upper portion of the curve by using, for example, the unloading method \([1]\) (in which the first variation is accomplished, but without the P vs \(l\) curve) will only change the value of the considered area of the P vs \(\Delta e\) curve.

For quite brittle materials such a construction has been used for a long time (see, in particular, \([2]\)) for determining the unit work for failure (the effective surface energy).

Treatment of the experimental results for compact 20-mm-thick samples of 12Kh2MFA steels with the use of the unloading method provided the P vs \(\Delta e\) curves shown in Fig. 3 \([1]\). The increase in crack length \((l_C - l_0)\), determined from the calibration curve of pliability \([1]\), was 2.3 mm for 12Kh2MFA steel and 2.9 mm for 15KhMFA steel. The corresponding values of the work for failure \(\Delta e\) are 264 and 378 kgf·mm and the parameter of fracture toughness \(KIC = \sqrt{E_G/\Delta e} = \sqrt{EA_{\Delta e}(l_C - l_0)}\) are 340 kgf/mm\(^{3/2}\) for 12Kh2MFA steel and 370 kgf/mm\(^{3/2}\) for 15Kh2MFA steel. It is characteristic that bond tests of samples with cross section of 100×100 mm of 15Kh2MFA steel provided a value of \(KIC\) of 370-380 kgf/mm\(^{3/2}\) \([3]\).

Therefore, the use of the P vs \(\Delta\) elastic displacement curve makes it possible to determine the fracture toughness \(KIC\) on small samples without special devices. This opens up the possibility of classifying the form of failure on the basis of the criterion \(\Delta p/\Delta e\). It should be noted that making the test conditions more severe...