\section*{\lambda\text{-}INVARIENTS AND \Gamma\text{-}TRANSFORMS}

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In this paper we give the relationship between the \lambda\text{-}invariants of two power series which are naturally associated to the \Gamma\text{-}transform of a \textit{p}\text{-}adic measure. Since the \textit{p}\text{-}adic \textit{L}\text{-}functions over \mathbb{Q} arise as such \Gamma\text{-}transforms, we obtain information about the "minus part" of Iwasawa's \lambda\text{-}invariant for the basic \textit{Z}\textit{p}\text{-}extension of an abelian CM\text{-}field.

\section*{1. \textit{p}\text{-}ADIC MEASURES}

Fix an odd prime, \textit{p}. Let \textit{C}\textit{p} denote the completion of an algebraic closure of \mathbb{Q}\textsubscript{\textit{p}}, and let ord denote its valuation, normalized by ord(\textit{p}) = 1. Let \textit{O} be the ring of integers in a finite extension of \mathbb{Q}\textsubscript{\textit{p}} in \textit{C}\textsubscript{\textit{p}}, with parameter \textit{\pi}. Recall that we may write \textit{Z}\textsubscript{\textit{p}}\times = \textit{V} \times \textit{U} where \textit{V} is the group of (\textit{p} - 1)st roots of unity in \textit{Z}\textsubscript{\textit{p}} and \textit{U} = 1 + \textit{pZ}\textsubscript{\textit{p}}. The projections onto \textit{V} and \textit{U} are denoted \omega and \langle \ldots \rangle, respectively. Also, fixing a topological

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generator $u$, of $U$, we have a group isomorphism, $\varphi : \mathbb{Z}_p \to U$, given by $\varphi(y) = u^y$.

Let $\Lambda$ denote the $\mathcal{O}$-valued measures on $\mathbb{Z}_p$. As is well-known, $\Lambda$ is a ring, (under convolution), which is isomorphic to the ring of formal power series, $\mathcal{O}[[T - 1]]$. Explicitly, for $x \in \mathbb{Z}_p$, let

$$ T^x = \sum_{n=0}^{\infty} \binom{x}{n} (T - 1)^n \in \mathcal{O}[[T - 1]] . $$

The power series associated to $\alpha \in \Lambda$ is then defined by

$$ \hat{\alpha}(T) = \int_{\mathbb{Z}_p} T^x \, d\alpha(x) = \sum_{n=0}^{\infty} b_n(\alpha)(T - 1)^n $$

(1) where $b_n(\alpha) = \int_{\mathbb{Z}_p} \binom{x}{n} \, d\alpha(x)$.

Mapping $\alpha \to \hat{\alpha}(T)$, we obtain a ring isomorphism between $\Lambda$ and $\mathcal{O}[[T - 1]]$.

A classical theorem of Mahler states that any continuous function $f : \mathbb{Z}_p \to \mathbb{Q}_p$ may be written uniquely in the form

(2) $$ f(x) = \sum_{n=0}^{\infty} a_n(f) \binom{x}{n} , $$

where $a_n(f) \in \mathbb{Q}_p$, $a_n(f) \to 0$ as $n \to \infty$. In fact

$$ a_n(f) = \sum_{j=0}^{n} (-1)^{n-j} \binom{n}{j} f(j) . $$

This theorem may be generalized to continuous functions.