STABILITY OF EINSTEIN–HERMITIAN VECTOR BUNDLES

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Einstein–Hermitian vector bundles are defined by a certain curvature condition. We prove that over a compact Kähler manifold a bundle satisfying this condition is semistable in the sense of Mumford–Takemoto and a direct sum of stable Einstein–Hermitian subbundles.

1. Introduction

In his paper [6] Kobayashi introduced the notion of an Einstein–Hermitian vector bundle, i.e. a bundle satisfying a certain curvature condition (Einstein condition). This condition is (over a compact complex manifold) sufficient for the bundle to be semistable in the sense of Bogomolov [1], as Kobayashi proved in [6]. In his recent paper [4] Kobayashi announces the following

THEOREM. An Einstein–Hermitian vector bundle over a compact Kähler manifold is semistable in the sense of Mumford–Takemoto and a direct sum of stable Einstein–Hermitian subbundles.
The purpose of this note is to give a proof of this Theorem *). Therefore we at first recall the definition and some properties of Einstein-Hermitian bundles and state a result on the first Chern forms of an Einstein-Hermitian bundle and a subbundle (Proposition 1), which is proved by purely differential geometric methods.

Next we define a differential geometric version of the (semi-)stability in the sense of Mumford-Takemoto and prove the first part of the Theorem: An Einstein-Hermitian vector bundle is semistable (Proposition 2). The main tool of the proof is a vanishing theorem of Kobayashi for sections in Einstein-Hermitian bundles.

If an Einstein-Hermitian bundle $E$ is not stable, we get by Proposition 1 a splitting of $E$ outside an analytic subset of the base manifold, which we extend to a global splitting by sheaf theoretic arguments. This leads to a splitting of $E$ into stable subbundles (Proposition 4).

Although stable bundles and Einstein-Hermitian bundles share many common properties, it is not known until now if every stable bundle satisfies the Einstein condition.

2. Einstein-Hermitian vector bundles

We start with some general facts on Einstein-Hermitian bundles; details can be found in Kobayashi's papers [4], [5],[6] or in [7],[8].

*) I communicated this proof to Prof.Kobayashi, and he wrote me that his proof is in essential parts different from mine. He kindly suggested a publication of my proof because his will not be published "probably for another year or so"