REPRESENTATION OF DISTRIBUTIONS WITH COMPACT SUPPORT

Richard D. Carmichael and Winston W. Walker

Distributions having compact support are represented as the boundary value of Cauchy and Poisson integrals corresponding to tubular radial domains $T^C$ in $\mathbb{C}^n$ where $C$ is an open convex cone. The Cauchy integral of $U \in \mathcal{E}'$ is shown to be an analytic function in $T^C$ which satisfies a certain boundedness condition. All analytic functions in $T^C$ having this boundedness condition have a distributional boundary value which can be used to determine an $\mathcal{E}'$ distribution. The results are extended to vector valued distributions.

1. INTRODUCTION

Köthe [10] has proved that a generalized function defined on a closed curve in the extended complex plane has a representation as a certain limit of two functions, one analytic in the interior of the domain bounded by the curve and the other analytic in the exterior. Tillmann [19] generalized Köthe's theory to unbounded domains in $\mathbb{C}^1$ and to functions of several variables defined on regions in $\mathbb{C}^n$ that are the product of unbounded domains in the plane; in particular Tillmann's theorems apply to half planes in $\mathbb{C}^1$ and to octants in $\mathbb{C}^n$.

In [19] Tillmann obtained a characterization of the analytic functions which represent the distributions of compact support, the $\mathcal{E}'$ distributions. Several other authors have been concerned with representing $\mathcal{E}'$; we note Bremermann and Durand [4], Bremermann [2,3], and Mitrović [11,12]. Except for Tillmann, these authors
work in the setting of half planes or octants; and
Tillmann's results apply most readily to these domains.

Vladimirov has introduced the concept of the repre-
sentation of distributions as boundary values of func-
tions analytic in tube domains. In [23] he character-
izes functions which are analytic in tube domains and
which have boundary values in $\mathcal{D}'$, the space of tempered
distributions, on the distinguished boundary of the tube
domain. Vladimirov's results are generalizations of
theorems previously obtained by Tillmann [21] for oc-
tants and by Vladimirov [24] for tube domains corres-
dponding to the forward and backward light cones.

In the present paper the distributions of compact
support are represented as boundary values of functions
defined in general tube domains in $\mathcal{C}^n$. We obtain two
separate representations of $\mathcal{E}'$, one using the Cauchy
integral and the other using the Poisson integral of
elements in $\mathcal{E}'$ corresponding to the tube domains. We
give necessary and sufficient conditions for analytic
functions in tube domains to represent elements in $\mathcal{E}'$.
Further, we generalize our results to vector valued dis-
tributions. It will be seen that several of the results
presented in this paper are considerable generaliza-
tions of theorems obtained by the above authors who have
studied the problem of representation of $\mathcal{E}'$ and $\mathcal{D}'$
distributions.

2. NOTATION AND DEFINITIONS

The n dimensional notation to be used in this paper
will be the same as in Carmichael [5, p.844]. $x,y,t,\eta$