Scattering of antiplane shear waves by a finite crack in piezoelectric laminates

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Summary. Following the dynamic theory of linear piezoelectricity, we consider the scattering of horizontally polarized shear waves by a finite crack in a composite laminate containing a piezoelectric layer. The piezoelectric layer is bonded between two half-spaces of a different elastic solid. The crack is normal to the interfaces and is placed at an equal distance away from them. Both cases of a partially broken layer and a completely broken layer are studied. Fourier transforms are used to reduce the problem to the solution of a pair of dual integral equations. The solution of the dual integral equations is then expressed in terms of a singular integral equation. The propagation of symmetric first mode is studied numerically, and the dynamic stress intensity factor and the dynamic energy release rate are obtained for some piezoelectric laminates.

1 Introduction

In the past several years, researchers have begun using piezoelectric devices for structural vibration suppression. For the purposes of active vibration and noise control, piezoelectric devices have shown great potential as sensors or actuators, because they are inexpensive, small, and lightweight, and can be easily bonded to or into any structure [1], [2]. In some of these applications, a major concern in practical operations has been mechanical and electric failure of the piezoelectric layers. Fracture can lead to undesirable mechanical and dielectric responses for these advanced piezoelectric materials. Recently several researchers have performed analyses of piezoelectric materials containing defects. Sosa and Khutoryansky [3] addressed the problem of an elliptic hole embedded in a piezoelectric solid within the framework of in-plane electroelastic interactions. Li and Mataga [4], [5] performed an analysis for the transient response of a semi-infinite, anti-plane crack propagating in a hexagonal piezoelectric medium. They obtained a closed form solution for constant speed crack propagation under external anti-plane shear loading with the conducting electrode type of electric boundary condition imposed on the crack surface [4]. They also made an alternative assumption that between the growing crack surfaces there was a permeable vacuum free space, in which the electrostatic potential was nonzero [5]. Shindo et al. [6] considered the problem of determining the singular stress and electric fields in an orthotropic piezoelectric ceramic strip containing a Griffith crack under longitudinal shear and studied the effects of crack face boundary conditions on the fracture mechanics of piezoelectric solids. Narita and Shindo [7] also carried out an analysis of the scattering of horizontally polarized shear waves by a finite crack in an orthotropic piezoelectric ceramic. However, electroelastic behavior of laminated piezoelectric composite structures with cracks has received very little attention despite the fact that
many piezoelectric devices are constructed in laminated form. Accordingly, there is a need to investigate the electroelastic fracture mechanics analysis of laminated piezoelectric structures.

In this paper, following the dynamic theory of linear piezoelectricity, we consider the scattering of horizontally polarized shear waves by a finite crack in a piezoelectric composite laminate. The piezoelectric layer is bonded between two half-spaces of a different elastic solid. The crack is normal to and bisected by the midplane of the piezoelectric layer. Both the partially broken layer and the completely broken layer are studied. Fourier transforms are used to reduce the mixed boundary value problem to the solution of a pair of dual integral equations. These equations are further reduced to a singular integral equation [8]. The propagation of the first symmetric mode is studied numerically, and the dynamic stress intensity factor and the dynamic energy release rate are obtained for several values of frequency, geometrical parameters, and piezoelectric material constants. It is noteworthy that the transition from a partially broken layer to a completely broken layer appears smooth on these curves, although this is to be expected on physical grounds.

2 Problem formulation

The piezoelectric layered composite shown in Fig. 1 is composed of a single piezoelectric layer of width 2h with the elastic stiffness constant $c_{44}$ measured in a constant electric field and the mass density $\rho$ that is bonded to two half-spaces having elastic properties $c_{E}$ and $\rho^E$. A set of Cartesian coordinates $(x, y, z)$ is attached to be the center of the crack for reference purposes. A crack of length 2c is situated on the $x$ axis so that it is bisected by the $y$ axis and is normal to the interfaces. The piezoelectric layer is poled in the $z$-direction exhibiting transversely isotropic behavior (hexagonal symmetry). Let time-harmonic antiplane shear waves originating at $y = \infty$ be incident normally on the crack. Quantities in the neighboring half-spaces will subsequently be designated by a superscript $E$.

The boundary value problem simplifies considerably if only the out-of-plane displacement and the in-plane electric fields are treated, i.e.

$$u_x = u_y = 0, \quad u_z = u_z(x, y, t),$$

$$E_z = E_z(x, y, t), \quad E_y = E_y(x, y, t), \quad E_x = 0,$$

$$u_x^E = u_y^E = 0, \quad u_z^E = u_z^E(x, y, t),$$

where $(u_x, u_y, u_z)$ and $(E_x, E_y, E_z)$ are the components of displacement and electric field vectors. The constitutive equations can be written as

$$\sigma_{xz} = c_{44}u_{z,x} - \varepsilon_{15}E_x,$$

$$\sigma_{yz} = c_{44}u_{z,y} - \varepsilon_{15}E_y,$$

$$D_x = \varepsilon_{15}u_{z,x} + \varepsilon_{11}E_x,$$

$$D_y = \varepsilon_{15}u_{z,y} + \varepsilon_{11}E_y,$$

$$\sigma_{xx}^E = \varepsilon_{44}u_{z,x}^E,$$

$$\sigma_{yy}^E = \varepsilon_{44}u_{z,y}^E,$$

where $\sigma_{zz}, \sigma_{yy}$ and $D_x, D_y$ are the components of the stress tensor and electric displacement vector, $\varepsilon_{11}$ is the dielectric constant measured at constant strain, $\varepsilon_{15}$ is the piezoelectric con-