Wave scattering from inhomogeneous anisotropic layers

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Summary. An analytic method is outlined to solve the problem of the scattering of harmonic waves from an inhomogenous anisotropic layer which admits a plane of material symmetry. Horizontally and vertically polarized waves are represented by couples of backward and forward modes. The amplitude and the polarization of each couple are obtained via a first-order Riccati differential equation while continuity requirements, imposed on each couple at the edge of the layer, yield the necessary boundary conditions. Reflection coefficients are derived at the outset and within the layer as a function of the depth, and transmission of the energy flux is evaluated. A wave-splitting is introduced in a natural way, and comparison with previous investigations is performed.

1 Introduction

The propagation of harmonic or transient waves in discretely or continuously layered media takes a relevant place in various fields of theoretical and applied research such as geophysics, acoustics and electromagnetism. Direct and inverse problems are widely investigated in order to predict scattering properties of simple or composite layered materials and to infer their constitutive structure. Most of the contributions to these topics have been devoted to isotropic elastic or viscoelastic media [1], [2] as well as isotropic dispersive dielectrics [3]. Recently, a notable effort has been done at performing the solution of the more realistic case of anisotropic layered media [4] – [8]. Although the mathematical tools used for anisotropic media are the same as those for the isotropic case, some difficulties arise from the occurrence of the layer inhomogeneity and the non-scalar character of the governing differential equations. For instance, the evaluation of the so called propagator matrix is complicated by the assumption of continuously stratified layers and this motivates the study of homogeneous multilayered or periodically layered media which admit solutions in the form of plane waves [4], [5]. Concerning to electromagnetic waves, scattered from inhomogeneous anisotropic dielectrics, a wave-splitting technique has been used in order to obtain a non-linear differential equation for the matrix reflection coefficient (reflectivity) of transient dispersive waves [7], [8]. In the case of harmonic waves the reflectivity matrix obeys a Riccati-type equation [9]. The solution of these equations under suitable initial and boundary conditions is the main goal of the problem at hand. As shown in the present paper, a similar approach is also possible for the case of harmonic acoustic polarized on planes perpendicular to the stratification. However, our approach differs from the usual wave-splitting technique. In fact, the motivation of this work is primarily to look for a suitable physical description of the field inside the layer in terms of a displacement amplitude and a function which describes its polarization. In this respect, our task is not only to evaluate the reflection coefficient but, at the same time, to work out an explicit representation of
the wave field. The content of the papers is as follows. Section 2 is devoted to set up the model by introducing the constitutive restrictions for a solid layer which admits a plane of material symmetry and whose parameters are functions of the layer depth. As shown in Section 3, these restrictions allow us to decouple the governing differential equation into a scalar equation for horizontally polarized waves and a vector equation for vertically polarized waves. The two types of polarized waves are investigated separately in Section 4. Solutions for both cases are developed on the basis of the causality principle. In particular, vertically polarized waves are supposed to consist of two parts whose continuity is separately imposed at the edge of the layer. Polarization of each part, as function of the depth, is shown to satisfy a first-order Riccati differential equation. Reflection and transmission coefficients are then worked out from imposing continuity of the displacement and of the traction at the outset and used to evaluate the wave amplitude inside the layer. The thin layer limit is also performed. Transmission of the energy flux is then outlined, in Section 5, for the model at hand. Finally, Section 6 is addressed to the definition of the reflectivity as a function of the depth in the layer. Equivalence of our approach to known techniques [1], [10] is shown for the scalar case and a wave-splitting is proposed for the vertically polarized field.

2 Inhomogeneous layers with planes of reflective symmetry

Let us consider an anisotropic and inhomogeneous solid layer $\mathcal{L}$, bounded by two plane parallel surfaces $S_1$ and $S_2$ which divide it from two homogeneous, anisotropic solid media $\mathcal{B}_1$ and $\mathcal{B}_2$ respectively. Denoting by $h$ the thickness of the layer, we choose a set of Cartesian coordinates $(x, y, z)$ such that $\mathcal{B}_1$ and $\mathcal{B}_2$ be represented respectively by $x = 0$ and $x = h$ (see Fig. 1). Mechanical properties of $\mathcal{L}$, $\mathcal{B}_1$ and $\mathcal{B}_2$ are modelled via a linear theory by introducing a viscoelastic fourth-rank tensor $G \in \text{Lin}(\text{Sym})$ which, in general, is supposed to depend on $x = (x, y, z)$ and on time $t$. The Cauchy stress is then given by (see e.g. [11])

$$T(x, t) = \int_0^\infty G(x, s) \dot{e}(x, t - s) \, ds,$$

(2.1)

where $\dot{e} = \frac{1}{2} [\nabla u + (\nabla u)^T]$ is the infinitesimal strain tensor and $u$ is the mechanical displacement. The tensor $G$ is assumed to be non-singular and to satisfy the following symmetries:

$$G_{ijkl}(x, s) = G_{jikl}(x, s) = G_{ikjl}(x, s),$$

(2.2)

at any $x \in \mathbb{R}^3$, $s \in \mathbb{R}^+$. Looking at continuously stratified layers, $G$ will be taken as a function of $x$ in $\mathcal{L}(x \in [0, h])$ and independent on $x$ in $\mathcal{B}_1$ ($x < 0$) and $\mathcal{B}_2$ ($x > h$).

![Fig. 1. Model of the inhomogeneous layer](image)