Effect of Rigid Inclusions in Griffith Cracks in an Infinite Transversely Isotropic Medium

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With 5 Figures

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Summary — Zusammenfassung

Effect of Rigid Inclusions in Griffith Cracks in an Infinite Transversely Isotropic Medium. The problem of finding the stress distribution in an infinite transversely isotropic solid containing a Griffith crack is considered under conditions of plane strain. The state of stress is supposed to be due to a rigid inclusion which determines the crack profile over a specified region. It is assumed that the geometrical shape of the inclusion and its material properties are such as to permit a frictionless and complete fit. Quantities of physical interest, i.e. stress intensity factor and crack energy are derived and numerical results are given.

1. Introduction

In this paper we consider the problem of finding the stress distribution in an transversely isotropic infinite solid containing a Griffith crack subject to the assumptions pertinent to plane strain. The state of stress is supposed to be due to a rigid inclusion which determines the crack profile over a specified region. It is assumed that the geometrical shape of the inclusion and its material properties are such as to permit a frictionless and complete fit. The corresponding problem of an elastic inclusion is of much greater complexity because the equilibrium boundary itself is unknown and has to be determined. The present investigation is motivated by the consideration that the qualitative information derived for the simplified model here will give an approximate description of the solution of the more realistic problem of our elastic inclusion. It will also serve as an approximation where the geometry does not permit a snug contact.

The earliest investigation of the behaviour of solid structures containing cracks subject to tensile load seems to be due to INGLIS [1] whose results were
used by Griffith [2] for determining the fracture strength of brittle materials. Griffith’s criterion, that the decrease in the strain energy must be balanced by a corresponding increase in the potential energy due to surface tension corresponding to the new surface area gained, is commonly used for determining the critical pressure beyond which the crack is unstable. It seems that the state of stress in almost all theoretical investigations so far published has been assumed to be either due to an internal pressure in the crack or to a tensile or a shearing load on the boundary. It is well known that these elastostatic boundary value problems may be reduced to a pair of dual integral equations whose solution then leads to the biharmonic Airy stress function. The same mathematical technique is applicable where a rigid inclusion is inserted in a crack, as this results in a prescribed displacement over a part of the crack surface, the other part remaining free of stress.

2. Basic Equations

Under the assumption of plane strain, the stress-strain relations are [3]:

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} = 
\begin{bmatrix}
c_{11} & c_{12} & 0 \\
c_{12} & c_{22} & 0 \\
0 & 0 & c_{66}
\end{bmatrix} 
\begin{bmatrix}
e_x \\
e_y \\
v_{xy}
\end{bmatrix},
\]

where \(c_{11}, c_{12}, c_{22}\) and \(c_{66}\) denote the elastic constants.

In the absence of body force, using (2.1), the equilibrium equations can be expressed in terms of the displacements as follows:

\[
\begin{align*}
c_{11}u_{,xx} + c_{66}u_{,yy} + (c_{12} + c_{66}) v_{,xy} &= 0, \\
c_{66}v_{,xx} + c_{22}v_{,yy} + (c_{12} + c_{66}) u_{,xy} &= 0,
\end{align*}
\]

where \(u, v\) mean the displacements in the \(x\)- and \(y\)-axis direction and \(,()\) denotes differentiation with respect to the following variables. We apply the Fourier transform [4] with respect to \(x\) to (2.2) as follows

\[
\begin{align*}
-c_{11}\xi^2\widehat{u} + c_{66}\widehat{u}_{,yy} - i\xi(c_{12} + c_{66}) \widehat{v}_{,y} &= 0, \\
-c_{66}\xi^2\widehat{v} + c_{22}\widehat{v}_{,yy} - i\xi(c_{12} + c_{66}) \widehat{u}_{,y} &= 0,
\end{align*}
\]

where

\[
\begin{align*}
\overline{u}(\xi, y) &= \int_{-\infty}^{\infty} u(x, y) e^{i\xi x} \, dx, \\
\overline{v}(\xi, y) &= \int_{-\infty}^{\infty} v(x, y) e^{i\xi x} \, dx, \\
u(x, y) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{u}(\xi, y) e^{-i\xi x} \, d\xi, \\
v(x, y) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{v}(\xi, y) e^{-i\xi x} \, d\xi.
\end{align*}
\]