Multiple plate problem in potential two-dimensional flow

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Summary. Presented is an elementary solution which is a particular solution of a thin slit in the potential two-dimensional flow. The solution satisfies the following conditions: 1) the remote velocities are equal to zero, 2) the normal velocity of fluid to the thin slit is a Dirac function, in other words, a flow with unit magnitude is passing through some point at the slit surface, 3) Joukowski hypothesis is used. The velocity at leading edge of the slit is infinite and at trailing edge is finite. After using the obtained elementary solution, the multiple plate problem in potential two-dimensional flow can be reduced to a system of Fredholm integral equations. Numerical examples are given to demonstrate the use of proposed approach.

1 Introduction

The problem of potential two-dimensional flow (abbreviated as PTDF) has history of more than a hundred of years. However, the solution of multiple plate problem in PTDF cannot be obtained easily. In an earlier suggested approach for the thin airfoil problem, the density of velocity circulation along the thin airfoil is used to be the unknown function and therefore a singular integral equation for the problem is obtained [1]. For the crack problem in antiplane elasticity is similar to the problem in PTDF, so same thing happens. Most of the investigators reduced the antiplane crack problem to a singular integral equation [2]. On the other hand, many powerful methods have been suggested in the field of computational fluid mechanics [3], [4]. However, we feel that the finite element method is not convenient to solve the potential flow problem of an infinite region containing many plates.

More recently, a new approach is proposed to solve the multiple crack problem of antiplane elasticity, and the problem is reduced to a solution of Fredholm integral equation [5]. In this paper, the Fredholm integral equation approach is extended to the solution of multiple plate problem in PTDF.

As pointed out in [6], the choice of the undetermined function and known function in the integral equation will determine the structure of kernel of this equation. In fact, the kernel in the proposed integral equation is closely related to an elementary solution cited below. The elementary solution satisfies the following conditions: 1) the remote velocities of fluid are equal to zero, 2) the normal velocity of fluid to the thin slit is a Dirac function, in other words, a flow with unit magnitude is passing through some point at the slit surface, 3) Joukowski hypothesis is used, that is to say, the velocity at the leading edge of the slit is infinite and at the trailing edge is finite. By the use of the obtained elementary solution, the multiple plate problem in PTDF can be reduced to a system of Fredholm
integral equations of second kind. Finally, numerical examples are given to demonstrate the use of proposed approach. In the examples, the forces and moment applied on the plate are calculated. Clearly, the obtained numerical results will be useful for aircraft design.

2 Formulation of the multiple plate problem in PTDF

It is well known that [1], in two-dimensional incompressible potential flow, the complex potential \( w(z) \) can be expressed as

\[
w(z) = \varphi(x, y) + i \psi(x, y),
\]

where \( \varphi(x, y) \) being the potential function and \( \psi(x, y) \) being the stream function. Both two functions \( \varphi(x, y) \) and \( \psi(x, y) \) are harmonic functions. Also, the velocity components \( u \) (in \( x \)-direction) and \( v \) (in \( y \)-direction) will be written by

\[
w'(z) = u - iv
\]

or

\[
\begin{align*}
  u &= \frac{\partial \varphi}{\partial x} - i \frac{\partial \psi}{\partial y}, \\
  v &= \frac{\partial \varphi}{\partial y} + \frac{\partial \psi}{\partial x}.
\end{align*}
\]

In order to obtain the elementary solution, it is assumed that a flat slit/crack is placed in an infinite continuum (see Fig. 1). In addition, at the point \( z = s \) the velocity component \( v \) becomes infinite and along the remainder part of the flat slit \( v \) will be zero. Thus, we propose the following boundary condition

\[
v \bigg|_{z=0^\pm} = \frac{\partial \varphi}{\partial y} \bigg|_{z=0^\pm} = -\delta(x - s), \quad |x| < a.
\]

In addition, the following conditions are also assumed

1. \( w'(z) \) approaches to zero as \( z \to \infty \).
2. \( w'(z) \) becomes infinite at point \( A \) and finite at point \( B \) in Fig. 1.

It is easy to verify that, by the use of a previously obtained solution for the antiplane crack problem [5], the investigated solution in our case is obtainable

\[
w'(z) = u - iv = \frac{Y(s)}{Y(z)(z - s)},
\]

where

\[
Y(z) = \left(\frac{z + a}{z - a}\right)^{1/2}
\]