A Transversely Isotropic Half-Space Indented by a Flat Annular Rigid Stamp in the Presence of Adhesion

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With 3 Figures

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1. Introduction

In recent years, the interest in punch problems has grown appreciably in connection with various technical applications. The problem, in which the stamp is a flat-ended cylinder, has important applications in Soil Mechanics [1], especially in the theory of safety of foundations e.g. punch can be taken as a pillar erected on a layered soil. Numerous investigations have been carried out on this line [2]–[7].

The actual contact problem must be analyzed in the consideration of friction on the interface between stamp and elastic body. However, because the friction is affected by the properties of the body, the surface conditions, etc., it is difficult to determine the magnitude of the friction. Therefore, we consider usually the following two cases in the theoretical analysis of the problem, (1) stamp problem without friction, and (2) the problem in the presence of adhesion. Generally, the analysis in the second case becomes more complicated than that in the first case.

In the previous paper [8], we analyzed the first case of the axisymmetric problem on an transversely isotropic half-space indented by a rigid annular stamp. In the present paper we deal with the second problem.

2. Basic Equations of Elasticity

Denote cylindrical coordinates of a point by \((r, \theta, z)\) where the Z-axis coincides with the material axis of symmetry. For a symmetric deformation, the non-vanishing components of displacement and stress are denoted by \((u_r, u_z)\) and \((\sigma_{rr}, \sigma_{zz}, \sigma_{r\theta}, \sigma_{\theta z})\) respectively. The displacements and stress which satisfy the equilibrium equations are expressed in terms of two potential functions \(\phi_1(r, z)\) and \(\phi_2(r, z)\) that are harmonic in their respective \((r, \theta, z)\) spaces. These expressions as given in Green and Zerna [9] are.

\[
u_r = \frac{\partial}{\partial r} [\phi_1(r, z_1) + \phi_2(r, z_2)]
\]

\[
u_z = \frac{k_1}{v_1} \frac{\partial}{\partial z_1} \phi_1(r, z_1) + \frac{k_2}{v_2} \frac{\partial}{\partial z_2} \phi_2(r, z_2)
\]
\[ \sigma_{zz} = C_{44} \left[ (1 + k_1) \frac{\partial^2}{\partial z_1^2} (r, z_1) + (1 + k_2) \frac{\partial^2}{\partial z_2^2} \phi_2(r, z_2) \right] \]  
\[ \sigma_{rz} = C_{44} \left[ \frac{(1 + k_1)}{\sqrt{v_1}} \frac{\partial^2}{\partial r \partial z_1} \phi_1(r, z_1) + (1 + k_2) \frac{\partial^2}{\partial r \partial z_2} \phi_2(r, z_2) \right]. \]  

The other components may be similarly expressed. The dimensionless parameters \( v_\alpha \) and \( k_\alpha (\alpha = 1, 2) \) are functions of elastic constants \( C_i \); i.e. \( v_1 \) and \( v_2 \) are the roots of the equation.

\[ C_{44} C_{11} v^2 + [C_{13}(2C_{44} + C_{13}) - C_{44} C_{23}] v + C_{33} C_{44} = 0, \]  
and \( k_1 \) and \( k_2 \) are defined by

\[ k = \frac{C_{11} v_\alpha - C_{44}}{C_{13} + C_{44}}, \quad \alpha = 1, 2. \]  

3. Formulation of the Problem

Consider an infinite transversely isotropic half-space occupying the region \( z \geq 0 \), and let the annular stamp be adhered to the portion \( z = 0, a \leq r \leq b \) where \( a \) and \( b \) are the inner and outer radii of the stamp, as shown in Fig. 1.

\[ \sigma_{zz}(r, 0) = \sigma_{rz}(r, 0) = 0, \quad 0 \leq r \leq b, \]  
\[ u_r(r, 0) = 0, \quad a \leq r \leq b, \]  
\[ u_z(r, 0) = \varepsilon_0, \quad a \leq r \leq b. \]  

In addition, all stresses and displacements vanish at infinity. The appropriate potential functions satisfying the boundedness conditions are given by:

\[ \phi_1(r, z_1) = \int_0^\infty \xi^{-1} A(\xi) re^{-\xi z_1} J_0(\xi r) \, d\xi, \]  
\[ \phi_2(r, z_2) = \int_0^\infty \xi^{-1} B(\xi) e^{-\xi z_2} J_0(\xi r) \, d\xi, \]