A Thermodynamic Yield Criterion in Viscoplasticity

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Summary — Zusammenfassung

A Thermodynamic Yield Criterion in Viscoplasticity. A phenomenological thermodynamic theory of viscoplasticity is presented in which the internal variables are the plastic deformation tensor and a hardening variable. The latter is defined in such a way that a yield criterion follows from the dissipation inequality if the free energy is an increasing function of the hardening variable. The limiting case of inviscid plasticity is explored, and the significance of the results is discussed in terms of dislocation kinematics.


1. Introduction

In the past decade there has occurred a remarkable convergence in the formulation of models for the inelastic behavior of solids. In particular, two concepts stand out as common to most recent theories. One is the notion, introduced by Lee and Liu [1], of an intermediate configuration which would be attained if a deformed material element could be unstressed elastically; accordingly the deformation-gradient tension $F$ is decomposed in the form

$$F = F_o F_p.$$  

(1)

The other feature is the renewed use of internal variables to characterize the inelastic thermodynamic state of a material element, as opposed to characterization by means of memory functionals. In particular, the tensor $F_p$, or at least its right Cauchy-Green tensor $C_p = F_p^2 F_p$, is included among the internal variables (a procedure apparently pioneered by Green and Tobolsky [2]). It has been pointed out by Mandel [3] that the inelastic rotation (the orthogonal factor in the polar decomposition of $F_p$) drops out of the constitutive equations only in certain special cases of isotropy, so that theories using $C_p$ rather than $F_p$ (e.g. [2], [4]—[7]) are limited to such cases.

The main differences among the various modern theories of plasticity lie in the
choice of the remaining internal variables. Three categories can be distinguished: (1) the general (e.g. [7]—[9]), in which these variables are scalar-valued and tensor-valued, vaguely called “work-hardening variables” or the like, with their number and nature left unspecified; (2) the structural (e.g. [10]—[14]), where these are interpreted in terms of dislocation densities, arrangements and the like, and where \( F_p \) appears in the constitutive functions only through \( F_e = F F_p^{-1} \), though \( F_p \) is still an internal variable in the sense that a rate equation for it exists; and (3) the phenomenological (e.g. [1], [4]—[6], [15]), where typically, as in classical plasticity theory, a single work-hardening variable \( \kappa \) is defined by a rate equation in such a way that its value can be determined from the history of stress and strain. In classical plasticity theory two commonly used definitions are (a) \( \dot{\kappa} = \text{tr} (T D_p) \) and (b) \( \dot{\kappa} = \sqrt{\text{tr} D_p^2} \), where \( T \) denotes stress and \( D_p \) plastic deformation rate. The two definitions are equivalent in the case of the isotropically hardening Mises material, but in general they are quite different, particularly in regard to their behavior at \( D_p = 0 \): while (a) is linear, (b) is nonlinear and consequently (since it is homogeneous of degree one in \( D_p \)) non-smooth. Modern theories of plasticity belonging to the third category, beginning with [4], invariably use of a linear definition equivalent to \( \dot{\kappa} = \text{tr} (A L_p) \), where \( L_p = F_p F_p^{-1} \) and \( A \) is a tensor-valued state function. In this paper the consequences of assuming a form that is a generalization of definition (b) above will be explored.

I have shown in a previous paper [16] that when a dissipation function (equivalent to the classical “rate of plastic work”) is taken as a positive definite non-smooth function that is homogeneous of the first degree in the internal-variable (e.g. plastic strain) rates, a yield-criterion necessarily results and need not be assumed. The approach in that paper was not formally thermodynamic in the sense that no thermodynamic potential was used, nor was the Clausius-Duhem inequality invoked. The purpose of the present paper is to present a formal thermodynamic theory of viscoplastic yielding. That is, a yield criterion of the usual form, but quite general, will be shown to follow from the Clausius-Duhem inequality when certain assumptions are made concerning the structure of the rate equations governing the internal variables \( F_p \) and \( \kappa \) — in particular, when \( \kappa \) is defined by a generalization of the aforementioned classical definition (b). This result is in contrast to all previous theories of viscoplastic behavior known to me, where a yield condition is assumed either to exist independently [1], [3]—[15] or not to exist at all [17].

2. Kinematics and Thermodynamics

The deformation gradient \( F \) is decomposed according to Eq. (1), where \( F_p \) transforms an infinitesimal line element from the reference configuration to the intermediate configuration, and \( F_e \) from the latter to the current configuration. The intermediate configuration is chosen to be isoclinic (as defined by Mandel [3]), so that \( F_p \) is unaffected by a superimposed rigid-body motion. Not only is then the “total” (right) Cauchy-Green tensor \( C = F^T F \) invariant, but so is the “elastic” Cauchy-Green tensor \( C_e = F_e^T F_e \), since

\[
C = F_p^T C_e F_p.
\]