Static stresses in a periodically layered anisotropic elastic composite containing a periodic array of planar cracks

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Summary. The problem of calculating the static elastic stresses in a periodically layered anisotropic composite containing a periodic array of planar cracks is considered. We formulate the problem in terms of a system of simultaneous finite-part singular integral equations which can be solved numerically using collocation techniques. The solution of the integral equations enables relevant quantities such as the stress intensity factors to be computed. Numerical results are obtained for specific cases of the problem.

1 Introduction

Ang [1], [2] and Clements [3] have examined the problem of determining the elastic stresses around a single planar crack in the interior of an anisotropic layer sandwiched between two anisotropic half-spaces. Using suitable Fourier integral representations for the displacements and the stresses, they reduced the problem to a system of three Fredholm integral equations of the second kind which were then solved numerically to compute the crack energy or the stress intensity factors.

In the present paper, we employ similar integral representations for the displacements and the stresses to formulate the problem of a periodically layered anisotropic composite containing a periodic array of planar cracks under static loading in terms of a system of simultaneous finite-part singular integral equations. These integral equations may be solved numerically by using collocation techniques described in Ioakimidis [4] and Kaya and Erdogan [5]. Once they are solved, relevant quantities of interest, such as the stress intensity factors, can be readily computed. Numerical results are obtained for specific cases of the problem.

2 Statement of the problem

Referred to an $Ox_1x_2x_3$ Cartesian coordinate system, consider an infinite elastic space consisting of infinitely many layers $|x_1| \geq 0, 2mh < x_2 < (2m + 2)h, |x_3| \geq 0, m = 0, \pm 1, \pm 2, \pm 3, \ldots$, where $h$ is a given positive real constant. The layers are alternately occupied by two different anisotropic elastic materials. The interior of the layer $4mh < x_2 < (4m + 2)h$ ($m = 0, \pm 1, \pm 2, \pm 3, \ldots$) contains $N$ pairs of coplanar cracks in the regions $a_k < |x_1| < b_k, x_2 = (4m + 1)h, |x_3| \geq 0, k = 1, 2, 3, \ldots, N$, where $a_k$ and $b_k$ are real numbers which are such that $0 < a_1 < b_1 < a_2 < b_2 < a_3 < \cdots < a_N < b_N$. 
The cracks are opened by the action of internal tractions which are independent of time and the coordinates $x_1$ and $x_2$. A perfect bond is assumed to exist between the materials in the different layers. The problem is to determine the (Cartesian) displacements $u_k$ and stresses $\sigma_{kj}$ in the layers.

3 Formal solution

In the layer $4mh < x_1 < (4m + 2) h$ ($m = 0, \pm 1, \pm 2, \pm 3, \ldots$), let the displacements and stresses be given respectively by

$$u_k^{4m+1} = \text{Re} \left\{ \sum_s A_{ks} \int_0^\infty \left[ F_s(\xi) \sinh \left( i\xi[z_a - (4m + 1) hr_a]\right) \right. \
+ H(x_a - (4m + 1) h) G_s^+(\xi) \exp \left( i\xi[z_a - (4m + 1) hr_a]\right) \
+ H(-x_a + (4m + 1) h) G_s^-(\xi) \exp \left( -i\xi[z_a - (4m + 1) hr_a]\right) \left. \right] d\xi \right\}, \quad (3.1)$$

and

$$\sigma_{kj}^{4m+1} = \text{Re} \left\{ \sum_s L_{ks} \int_0^\infty \left[ F_s(\xi) \cosh \left( i\xi[z_a - (4m + 1) hr_a]\right) \right. \
+ H(x_a - (4m + 1) h) G_s^+(\xi) \exp \left( i\xi[z_a - (4m + 1) hr_a]\right) \
- H(-x_a + (4m + 1) h) G_s^-(\xi) \exp \left( -i\xi[z_a - (4m + 1) hr_a]\right) \left. \right] i\xi d\xi \right\}, \quad (3.2)$$

where the latin subscripts take the values of 1, 2 and 3, the summation over the greek suffix is from 1 to 3, Re denotes the real part of a complex number, $H(x)$ is the Heaviside unit-step function, $i = (-1)^{1/2}$, $z_a = x_1 + r_a x_2$, $A_s, L_{ks}$ and $r_a$ are constants (obtained using the elastic moduli of the material in the layer) as defined in Ang [1] or Clements [3], and $F_s(\xi), G_s^+(\xi)$ and $G_s^-(\xi)$ are arbitrary functions yet to be determined.

In the uncracked layer $(4m + 2) h < x_1 < (4m + 4) h$ ($m = 0, \pm 1, \pm 2, \pm 3, \ldots$), the displacements and stresses are chosen respectively to be given by

$$u_k^{4m+3} = \text{Re} \left\{ \sum_s A_{ks}^0 \int_0^\infty X_s(\xi) \sinh \left( i\xi[z_a^0 - (4m + 3) hr_a^0]\right) d\xi \right\}, \quad (3.3)$$

and

$$\sigma_{kj}^{4m+3} = \text{Re} \left\{ \sum_s L_{ks}^0 \int_0^\infty X_s(\xi) \cosh \left( i\xi[z_a^0 - (4m + 3) hr_a^0]\right) i\xi d\xi \right\}, \quad (3.4)$$

where the superscript $O$ denotes that the relevant constants are obtained using the elastic moduli of the material in the uncracked layer, and $X_s(\xi)$ are arbitrary functions yet to be found.