On the asymmetry of elastic-plastic moduli for a class of pressure-dependent models in strain space

J. H. Lee, Fairbanks, Alaska

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Summary. For a general class of constitutive equations of plastic strain rates, which are normal to the loading surface in strain space, it is shown that for associated pressure-dependent plasticity the elastic-plastic moduli are nonsymmetric. The asymmetry is shown to be incompatible with the consequences of a work assumption due to Naghdi and Trapp.

1 Introduction

For associated plasticity in stress space, the plastic strain rate is assumed to be normal to the yield surface. In strain-space plasticity [1], [2], the primary variables are the total strains and the total plastic strains. Conceivably, one may postulate that the plastic strain rate is normal to the loading surface in strain space. This assumption had been used in [3] for the von Mises (pressure-independent) yield criterion with associated flow rules, and in [4] for a special class of (pressure-dependent) yield criterion with nonassociated flow rules. For these cases [3], [4], it can be shown that the resulting elastic-plastic moduli, $M_{ijkl}^{pp}$ (to be defined in (10)), are identical with the ones from stress-space formulation. However, it will be shown that, when the plastic strain rate is assumed to be normal to the loading surface in strain space, for a general class of pressure-dependent associated plasticity models, the elastic-plastic moduli $M_{ijkl}^{pp}$ become nonsymmetric. This asymmetry is in contrast to the general notion that associated plasticity, pressure-dependent or not, should yield symmetric elastic-plastic moduli. The present note addresses the asymmetry in light of the assumption of work inequality obtained in [5] which was further elaborated in [6]. Standard index notation is used where repeated indices are summed. Small deformation is assumed.

2 The asymmetry

A loading surface $g$ in strain space is assumed to exist with the functional dependence $g = g(\epsilon_{kit}, \epsilon_{kl}^p, \kappa)$ where $\epsilon_{kit}$ and $\epsilon_{kl}^p$ are the total and plastic strains, respectively, and $\kappa$ is the strainhardening parameter. Defining $\dot{g}$ as

$$\frac{\partial g}{\partial \epsilon_{mn}} \dot{\epsilon}_{mn},$$

(1)
the general constitutive equations for plastic strain rate in strain space [2], for loading 
\( g = 0, \dot{g} > 0 \), are
\[
\dot{\varepsilon}_{pl}^p = \dot{\lambda} \dot{g} \dot{g}.
\]  
(2)

We note that (2) is applicable for both associated and nonassociated plasticity.

Concentrating on associated plasticity, we consider the following special class of con-
stitutive equations for \( \varepsilon_{pl} \)
\[
\dot{\varepsilon}_{pl}^p = \dot{\lambda} \frac{\partial g}{\partial e_{kl}}.
\]  
(3)

That is, the plastic strain rate is assumed to be normal to the loading surface in strain 
space. Comparing (3) with (2), one has \( \dot{\lambda} = \dot{\lambda} \dot{g} \), and \( \dot{\lambda} = \dot{g} \partial g \partial e_{kl} \). The consistency equation 
states that
\[
\dot{\gamma} = \frac{\partial g}{\partial e_{mn}} \dot{e}_{mn} + \frac{\partial g}{\partial e_{mn}^p} \dot{e}_{mn}^p + \frac{\partial g}{\partial \varepsilon} \dot{\varepsilon} = 0
\]  
(4)

with
\[
\dot{\varepsilon} = C_{et} \dot{e}_{et}^p
\]  
(5)

where \( C_{et} \) is a function of \( e_{et}, e_{et}^p, \varepsilon \). Using (3), (4) and (5), the constitutive equations for 
the plastic strain rate becomes
\[
\dot{e}_{et}^p = \frac{\dot{\gamma}}{A + \Gamma \frac{\partial g}{\partial e_{et}}}; \quad A + \Gamma > 0
\]  
(6.1)

where
\[
A = \frac{\partial g}{\partial e_{mn} \frac{\partial g}{\partial e_{mn}}}, \quad \Gamma = -\frac{\partial g}{\partial \varepsilon} C_{mn} \frac{\partial g}{\partial e_{mn}}.
\]  
(6.2)

One further assumes that the total strain (\( e_{et} \)) can be decomposed into elastic (\( e_{et}^e \)) and 
plastic (\( e_{et}^p \)) components. The Hooke's law states
\[
\sigma_{ij} = M_{ijkl} e_{kl},
\]  
(7)

where \( M_{ijkl} \) are the elastic moduli and are defined by
\[
M_{ijkl} = \mu (\delta_{ij} \delta_{kl} + \delta_{ik} \delta_{jl}) + \left( K - \frac{2}{3} \mu \right) \delta_{ij} \delta_{kl},
\]  
(8)

where \( \mu \) is the shear modulus, \( K \) is the bulk modulus and \( \delta_{ij} \) is the Kronecker delta. Using 
(6) and the rate form of (7), one has
\[
\dot{\sigma}_{ij} = M_{ijkl}^e (\dot{e}_{kl} - \dot{\varepsilon}_{et}^p) - M_{ijkl}^p \left( \dot{e}_{kl} - \frac{\dot{\gamma}}{A + \Gamma \frac{\partial g}{\partial e_{et}}} \frac{\partial g}{\partial e_{et}} \right).
\]  
(9)

The elastic-plastic moduli \( M_{ijkl}^p \) are defined as
\[
\dot{\sigma}_{ij} = M_{ijkl}^{ep} \dot{e}_{et} = (M_{ijkl}^e - M_{ijkl}^p) \dot{e}_{et}.
\]  
(10)

Comparing (10) with (9) one has
\[
M_{ijkl}^p = \frac{M_{ijmn}^e}{A + \Gamma \frac{\partial g}{\partial e_{mn}} \frac{\partial g}{\partial e_{et}}},
\]  
(11)