Plane convective motion produced in a layer of viscous incompressible liquid heated from below is considered in the presence of periodic modulation of force of gravity. To solve the nonlinear nonstationary Navier–Stokes and heat-convection equations, the method of expanding the amplitude of the motion in a power series is used. The limits of the stability of the beam are determined (in first approximation), as well as the amplitude of the steady-state convective vibrations. A comparison is made with the results of a calculation by the grid method.

It is known [1–8] that the modulation of the gravitational field exerts a considerable influence both on the conductive stability of the equilibrium and on the convective motions that result from the instability. The greater part of [1–6] is devoted to an investigation of the stability of the equilibrium of the liquid in the linear formulation. An analysis of the nonlinear parametric convective oscillations was carried out only numerically [7,8]. In [9], a method is proposed for the analytic solution of nonlinear stationary convection equations; the problem is solved, however, in the limit of large Prandtl numbers, as a result of which the parametric effects proper drop out of the consideration. In the present papers, a modified method of power-law expansion of the amplitude is used to investigate the finite-amplitude parametric convection produced in a horizontal layer of liquid heated from below in the presence of modulation of the gravitational field. The method makes it possible to find the limits of the convective stability (first approximation) and determine the amplitude of the steady-state convective oscillations. It is seen from the results that, depending on the parameters of the problem, either a "soft" or a "hard" excitation of the convective oscillations is possible. The results are compared with previously obtained [7,8] numerical data.

1. Let a plane infinite horizontal layer of a viscous incompressible liquid execute harmonic oscillations along a vertical axis. We consider planar convective motion produced in such a layer when the latter is heated from below. The layer thickness is $a$, and the temperature at the lower boundary is taken to be the reference point, while that of the upper boundary is equal to $-\Theta$. The origin is chosen to be on the lower plane and the $y$ axis is directed vertically upward, while the $x$ axis is horizontal.

The oscillations of the layer lead to periodic modulation of the gravitational field by variable inertial forces. We write down the convection equations in terms of the stream function $\psi$ and the temperature $T$, determined from the equilibrium distribution $-\Theta(y/a)$. After changing over to dimensionless variables, we obtain

\begin{align}
(1 - \frac{\partial}{\partial t} - \Delta) \psi + G(1 + \Theta \cos 2\omega t) \frac{\partial T}{\partial x} &= L(\psi, \Delta), \quad G = \frac{g\Theta a^3}{\nu^2} \\
(1 - \frac{\partial}{\partial t} - \frac{1}{P} \Delta) T + \frac{\partial \psi}{\partial x} &= L(\psi, T), \quad P = \frac{\nu}{\nu} \\
L(f, \phi) &= \frac{\partial f}{\partial x} \frac{\partial \phi}{\partial y} - \frac{\partial f}{\partial y} \frac{\partial \phi}{\partial x}
\end{align}

Here $G$ is the Grashof number, $P$ is the Prandtl number, $2\omega$ is the dimensionless part of the modulation, and $\phi$ is the relative amplitude of the modulation [7]. The units in which the length, time, the stream function, and the temperature are measured are chosen to be, respectively, $a$, $a^2/\chi$, $\nu$, and $\Theta$. All subse-
quent calculations will pertain to the case easiest to analyze, namely, a layer with three boundaries, since the boundary conditions for equations (1.1) then take the form

\[ \psi = \frac{\partial^2 \psi}{\partial y^2} = 0, \quad T = 0 \quad (y = 0, \ y = 1) \]  

(1.2)

Following the method of L. V. Kantorovich [10], we reduce the problem (1.1), (1.2) to a system of nonlinear equations for the coordinate functions. Bearing in mind an investigation of only stationary vibration regimes, we use as the basis functions harmonics of the type \( \sin n \omega t \) and \( \cos n \omega t \). Then, depending on the periodicity, two types of expansion are possible:

for "half-integer" solutions with period \( 2\pi/\omega \)

\[
\psi = \psi_0(x, y) + \sum_{n=1}^{2N} \left[ \psi_{2n}(x, y) \cos n\omega t + \psi_{2n-1}(x, y) \sin n\omega t \right]
\]

\[
T = T_0(x, y) + \sum_{n=1}^{2N} \left[ T_{2n}(x, y) \cos n\omega t + T_{2n-1}(x, y) \sin n\omega t \right]
\]  

(1.3)

and for "integer" solutions with period \( 2\pi/\omega \)

\[
\psi = \psi_0(x, y) + \sum_{n=1}^{N} \left[ \psi_{2n}(x, y) \cos 2n\omega t + \psi_{2n-1}(x, y) \sin 2n\omega t \right]
\]

\[
T = T_0(x, y) + \sum_{n=1}^{N} \left[ T_{2n}(x, y) \cos 2n\omega t + T_{2n-1}(x, y) \sin 2n\omega t \right]
\]  

(1.4)

In this section we present, for concreteness, only the formulas for solutions of the integer type.

Substituting (1.4) in (1.1), multiplying by \( \sin 2n\omega t \) and \( \cos 2n\omega t \), and averaging over the modulation period \( \pi/\omega \), we arrive at a system of nonlinear equations for the coordinate functions

\[
- \Delta \psi_0 + G \frac{\partial T_0}{\partial x} + \frac{\partial G}{\partial x} \frac{\partial T_0}{\partial x} = K_0(\psi, \Delta \psi)
\]

\[
- \frac{1}{P} \Delta T_0 + \frac{\partial \psi_0}{\partial x} - K_0(\psi, T)
\]

\[
- \frac{2n\omega}{P} \Delta \psi_{2n} - \Delta \psi_{2n-1} + G \frac{\partial T_{2n-1}}{\partial x} + \frac{\partial G}{\partial x} \left( \frac{\partial T_{2n-1}}{\partial x} + \frac{\partial T_{2n-1}}{\partial x} \right) K_{2n-1}(\psi, \Delta \psi)
\]

\[
- \frac{2n\omega}{P} T_{2n} - \frac{1}{P} \Delta T_{2n-1} + \frac{\partial \psi_{2n-1}}{\partial x} = K_{2n-1}(\psi, T)
\]  

(1.5)

\[
\frac{2n\omega}{P} \Delta \psi_{2n-1} - \Delta \psi_{2n} + G \frac{\partial T_{2n-2}}{\partial x} + \frac{\partial G}{\partial x} \left( \frac{\partial T_{2n-2}}{\partial x} + \frac{\partial T_{2n-2}}{\partial x} \right) + \delta_{nt} \frac{\partial G}{\partial x} = K_{2n}(\psi, \Delta \psi)
\]

\[
\frac{2n\omega}{P} T_{2n-1} - \frac{1}{P} \Delta T_{2n} + \frac{\partial \psi_{2n}}{\partial x} = K_{2n}(\psi, T) \quad (n = 1, 2, \ldots, N)
\]

Here

\[
K_0(\psi, T) = L(\psi, T_0) + \sum_{n=1}^{2N} \frac{1}{2} L(\psi_n, T_n)
\]

\[
K_{2n-1}(\psi, T) = L(\psi_{2n}, T_{2n-1}) + L(\psi_{2n-1}, T_{2n}) + \sum_{k,l=1; k+l=n}^{2N} \frac{1}{2} \left[ -L(\psi_{2k}, T_{2l}) + L(\psi_{2l}, T_{2k}) \right]
\]

\[
K_{2n}(\psi, T) = L(\psi_{2n}, T_{2n}) + L(\psi_{2n}, T_{2n}) + \sum_{k,l=1; k+l=n}^{2N} \frac{1}{2} \left[ -L(\psi_{2k}, T_{2l}) + L(\psi_{2l}, T_{2k}) \right]
\]