FLOW OF A DUAL-TEMPERATURE PLASMA IN THE CHANNEL OF A DISK-MAGNETOHYDRODYNAMIC GENERATOR, TAKING ACCOUNT OF NONEQUILIBRIUM IONIZATION AND RECOMBINATION REACTIONS

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The establishment of a supersonic one-dimensional flow of a dual-temperature, partially ionized plasma is investigated in the channel of a disk-MHD generator, taking account of nonequilibrium ionization and recombination reactions. A detailed formulation of the problem is given in [1]; flows are considered in the absence of ionization and recombination reactions and in the case of equilibrium reactions.

1. We shall consider a supersonic, one-dimensional steady-state flow of a partially ionized, dual-temperature plasma consisting of argon with the addition of cesium. Suppose that all functions depend only on the radius r and are independent of the coordinate z and the angle \( \theta \). The solution will be found for \( r_1 > r \geq r^s \), where \( r^s \) is the initial radius, \( r_1 \) is the radius at which the Mach number becomes equal to unity. Nonequilibrium ionization and recombination reactions are taken into account. The viscosity and thermal conductivity of the plasma are not taken into account. The magnetic field \( H(0, 0, H) \) is oriented along the z axis and it is assumed that the magnetic Reynolds number \( R_m \ll 1 \).

The system of starting equations is written out in [1].

The system of equations (1.1) to (1.6) in [1], and the corresponding coefficients are written on the following assumptions:

\[ u_{at} = u_{ae} = u = u, \alpha = n_s / n_s \ll 1, \alpha \theta \ll 1, \epsilon \theta \ll 1, T_s = T = T \left( \theta = T_s / T \right) \]

The symbols are the same here as in [1].

It will be assumed that only the cesium is ionized. In calculating the collision frequency \( \nu_{\alpha\beta} \) of particles of type \( \alpha \) with particles of type \( \beta \), the collisions are assumed to be elastic; inelastic collisions (except ionization, which is considered in explicit form) are taken into account by the coefficient \( \delta \). The coefficient of recombination \( B \) has been assumed equal to \( 5.2 \times 10^{-23} \cdot T_e^{7/2} \) (deg \( \times 10^3 \)) [2]. The coefficient of ionization \( A \) is expressed in terms of \( B \) in the following way:

\[ A = B \left( n_s^2 / n_{at} \right) \]

The suffix \( s \) denotes that the given value is calculated by Saeh's formula.

2. The systems of equations (1.1) to (1.6) from [1], after simple transformations, reduces to a system of four differential equations for determining \( u, p, T_e, \) and \( n_e \). In dimensionless variables, these equations have the form

\[
\begin{align*}
\frac{du_*}{dr^*} + \frac{r_*}{K^*} \frac{dp_*}{dr^*} &= - \frac{v_*^2}{u_*^2} - S_0 u_*^r r^* (1 - N) \\
\frac{5}{2} p_* \frac{du_*}{dr^*} + \frac{3}{2} u_* \frac{dp_*}{dr^*} &= - \frac{3}{2} a^2 g * K^* \frac{dT_*}{dr^*} + a^2 g * K^* T_* \frac{dn_*}{dr^*} - \frac{5}{2} p_* \frac{u_*}{r^*} + S_0 u_*^r P_1 - J^2 a^2 P_2
\end{align*}
\]

\[
\frac{3}{2} \alpha^2 \theta n_e u_e (1 - N) \frac{dT_e}{dr} + \alpha^2 \theta K_e T_e \frac{dn_e}{dr} = S_{\alpha\theta} u_e P_1 - \left( \frac{3}{2} \alpha^2 T_e + J_e \right) \alpha^2 P_1 - v_e (\theta T_e - p_e u_e),
\]

\[
d\frac{u_e}{dr} + \frac{u_e \cdot dn_e}{n_e dr} = \frac{P_2}{n_e} - u_e.
\]

Equation (2.1) is the equation of motion of the plasma, Eq. (2.2) is the heat flux equation for the plasma as a whole, Eq. (2.3) is the heat flux equation for electrons, and Eq. (2.4) is the equation of continuity for electrons.

Here,

\[
r^* = \frac{r}{T}, \quad u^* = \frac{u}{u_e}, \quad p^* = \frac{p}{p_e}, \quad n_e, a_1, a_2 = \frac{n_e, a_1, a_2}{n_e, a_1, a_2}, \quad T^* = \frac{T}{T_e}
\]

\[
T_e^* = \frac{T_e}{T_e}, \quad a^* = \frac{n_e^*}{n_e^*}, \quad n_e^* = n_{a_1}^* + n_{a_2}^*, \quad \lambda = \frac{m_e u_e^2}{kT_e}, \quad \sigma^* = \frac{\sigma n_e^*}{m_e^*}
\]

\[
S = \frac{\alpha^2 H}{\rho u^* c}, \quad \gamma^* = \frac{\gamma^*}{n_e^* k}, \quad E_* = \frac{E_e}{Mu^*}, \quad K^* = \frac{\theta T_e}{n_e^* u_e^*}
\]

\[
j^* = \frac{j_{\alpha c}}{\sigma c u^* H^*}, \quad p^* = n_e^* kT_e + n_e^* kT_e, \quad \theta^* = \frac{T_0}{T_e}, \quad \lambda^* = \frac{\lambda^*}{n_e^*}
\]

\[
P_2 = \lambda^* \frac{n_e^*}{u_e^*} - B^* n_e^* - \frac{J^*}{kT_e}, \quad \lambda^* = \frac{n_e^*}{n_e^*} + \frac{a^*}{v_e^*}
\]

\[
\nu^* = \frac{3}{2} \gamma \lambda_{ea} \frac{n_e^* \xi}{\xi}, \quad \nu = \frac{n_e^* T_e^*}{\nu^*}, \quad \nu_{ea} = \frac{n_e^* \xi}{\nu^*}, \quad \nu_{el} = \frac{\xi}{1 + \chi \nu^* + \frac{n_e^*}{n_e^*} \xi^2}
\]

\[
\xi = 23.4 - 1.17 \log n_e + 3.45 \frac{T_e}{kT_e}^* - 1600
\]

\[
\nu^* = \frac{16}{3} \frac{1}{\sqrt{2}} n_{a_1}^* \left( \frac{kT_e}{n_m^*} \right)^{1/2} Q_{ea_1}^*, \quad Q_e = \frac{Q_{ea_1}}{Q_{ea_1}}, \quad \nu^* = \frac{n_e^* T_e^*}{\nu^*}, \quad \nu_{ea} = \frac{\lambda_{ea}^* \xi}{2 (kT_e)^*}
\]

where \( \xi \) is the Coulomb logarithm. Collisions between electrons, cesium atoms, argon atoms, and ions are taken into account. The following formulas are used to calculate the values of \( \nu_{ea} \) and \( \nu_{el} \):

\[
\nu_{ea} = \frac{16}{3} n_e^* \left( \frac{kT_e}{n_m^*} \right)^{1/2} Q_{ea_1}^*, \quad \nu_{el} = \frac{16}{3} \left( \frac{kT_e}{n_m^*} \right)^{1/2} Q_{ea_1}.
\]

The cross sections \( Q_{ea_1} \) were taken from [3]: \( Q_{ea_1} = 1.0 \cdot 10^{-16} \text{ cm}^2 \) and \( Q_{ea_2} = 400 \cdot 10^{-16} \text{ cm}^2 \).

The system of equations (2.1) to (2.4) was solved for the following boundary conditions: \( u^* = p^* = T_{e_0} = n_e^* = 1 \) when \( r^* = 1 \).

3. The problem being considered was solved on a computer. The initial concentration of electrons was computed by Sachs' formula. It was assumed, in all variants, that

\[
\delta = 2, \quad \theta^* = 1, \quad \gamma^* = 100 \text{ cm}, \quad H = 10^4 \text{ G}, \quad T^* = 2000^\circ \text{ K}.
\]

When calculating variants a, b, c, and d, the following combinations of parameters were taken:

<table>
<thead>
<tr>
<th>Variant</th>
<th>( \nu_{a_1} ) cm/sec</th>
<th>( a_2 )</th>
<th>( \nu_{a_2} ) cm/sec</th>
<th>( a_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1</td>
<td>10^5</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>2</td>
<td>10^5</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>1</td>
<td>2 \cdot 10^5</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>1</td>
<td>10^4</td>
<td>0.002</td>
<td></td>
</tr>
</tbody>
</table>

The lower suffixes 1 and 2 with the letters a, b, c, and d denote \( K^* = 0 \) and \( K^* = 0.83 \), respectively.