On the Hypoelastic-Idealplastic Constitutive Model

By

Ch. Tsakmakis and P. Haupt, Darmstadt, Federal Republic of Germany

With 1 Figure

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Summary

Recently, the hypoelastic-idealplastic solid under simple shear has been studied. As an unexpected result, the slope of the stress-strain curve at the beginning of the plastic range turned out to be negative. While all other assumptions remained the same, this effect was recognized for all objective stress rates, which have been used for the hypoelastic constitutive law.

The main objective of this paper is an interpretation of this remarkable result, where this interpretation should be given in an acceptable and rational form. To this end, we apply the constitutive assumption of the elastic-idealplastic body for comparison.

1. Introduction

In continuum mechanics, tensor-valued time rates are applied to formulate incremental stress-strain relations for a phenomenological representation of inelastic material properties. In view of their physical and mathematical implications, the problem of a definition of convenient tensor-derivatives has produced a most active discussion in the literature. This discussion was initiated by numerical results, which had been achieved by Nagtegaal and de Jong [1] for the particular case of homogeneous simple shear in the context of rate independent plasticity. In fact, similar results had been obtained earlier by Lehmann [2]; this paper has been ignored for a longer period of time. The result was the following: As a consequence of the application of the Jaumann stress rate the simple model of linear kinematic hardening implies an oscillating shear stress as a function of the shear strain. Analogous results have been communicated by Dienes [3] in the context of hypoelasticity. Detailed investigations of linear kinematic hardening in the context of large plastic deformations have been carried out by Lee, Mallet and Wertheimer [4] and Dafalias [5].

In their article, Nagtegaal and de Jong [1] tried to get improved results: They formulated a theory on the basis of the Jaumann derivative, but they added to
the linear kinematic hardening model a limiting term, which clearly is convenient
on physical reasons. Those models of nonlinear kinematic hardening have been
investigated by Fressengeas and Molinari [7] as well as by Haupt and Tsakmakis
[6]. In [6] it has been demonstrated, that the application of the Jaumann dif-
ferential operator leads to physically unrealistic stress-strain relations. In parti-
cular, this is true for all kinds of limiting terms, which have been investigated.

In order to avoid the complications resulting from the Jaumann derivative,
new definitions for tensor-derivatives have been introduced, which should imply
better results. We do not wish to analyze in detail the numerous proposals. How-
ever, it should be mentioned, that in comparison to hypoelasticity, in plasticity
all details become much more complicated, since additional effects occur, which
result from the decomposition of the deformation into elastic and inelastic parts.

A different procedure to establish incremental constitutive relations is the
utilization of dual variables. In fact, the idea of dual variables is a continuation of
earlier investigations made by McVean [8] and Hill [9]. The concept of dual
variables has been developed by Tsakmakis [10] (see also Haupt and Tsakmakis
[11]). As an example, the weighted Cauchy stress tensor $T$ and the Almansi strain
tensor $A$ form a pair of dual variables within the framework of general continuum
mechanics. Each pair of dual stress and strain measures involves, in a natural
way, unique time derivatives, which are likewise dual to each other. In particular,
special Oldroyd derivatives, defined by Eqs. (2.7) and (2.8) of Section 2, are
associated to $T$ and $A$. Finally, it should be mentioned, that all dual variables
are conjugate in the sense of [8] or [9], whereas the converse statement is not true
in general.

A formulation of a linear kinematic hardening model, which is based on the
dual Oldroyd derivatives of $T$ and $A$, implies monotonic stress-strain relations in
the case of simple shear: This has been demonstrated in [6] (see in particular
Eqs. (64), (65), (75) and (78) of that paper). In this sense, we consider the notion
of dual variables and derivatives to be a convenient method to represent material
properties within the general frame of continuum mechanics (for more detail
about this topic see [16]).

In the following, we will discuss the hypoelastic-idealplastic constitutive model.
This discussion will be based on the application of the concept of dual variables
and derivatives. The hypoelastic-plastic material body without hardening has been
analyzed by Moss [12]. Moss applied different objective derivatives of the Cauchy
stress tensor to the problem of simple shear. Starting from the same assumptions,
he found the remarkable result, that in all particular cases the curve of the shear
stress versus the shear angle has a negative slope, if the plastic range is entered.
In its physical interpretation this corresponds to a local instability of the material.
The unexpected occurrence of such material response effects — independent of
their order of magnitude — is principally indicating the existence of physically
inconsistent assumptions in the system of constitutive equations: In many cases
the mathematical complexity of the constitutive model can imply such pheno-