On the Development of Non-Newtonian Secondary Flows in Tubes of Non-Circular Cross-Section

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With 2 Figures

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Summary

The role that the normal stresses play in the generation of non-Newtonian secondary flows in straight tubes of non-circular cross-section is examined from a theoretical standpoint. It is proven independent of any constitutive assumptions that in order for these secondary flows to occur in the absence of body forces, the axial velocity must give rise to some non-zero difference in the transverse normal stresses. This necessary condition is utilized to demonstrate that the development of secondary flows in certain non-Newtonian fluids can be influenced by the precise structure of the time derivatives in the constitutive equations. In particular, it is shown that certain frame-indifferent convected time rates give rise to secondary flows whereas others do not.

1. Introduction

It has been known for quite some time that the pressure-driven flow of a non-Newtonian fluid through a straight tube of non-circular cross-section is not, in general, rectilinear (i.e., unidirectional along the axis of the tube) like its Newtonian counterpart. A secondary flow occurs in the transverse planes of the tube (see Fig. 1) which causes fluid particles to undergo a spiraling motion down the tube. This secondary flow occurs when the velocity field and pressure gradient are independent of the axial coordinate and, thus, exists independent of end effects. Ericksen [1] was the first to recognize the existence of such non-Newtonian secondary flows. He proved that in order to maintain a steady rectilinear flow of a non-Newtonian fluid through tubes of non-circular cross-section a suitable distribution of external body forces would, in general, have to be applied to the fluid. Shortly after this discovery, Green and Rivlin [2] calculated the structure of the secondary flow in an elliptic tube for a Reiner-Rivlin fluid. They postulated that these results would be qualitatively correct for more general
non-Newtonian fluids which has, subsequently, been proven to be the case (c.f., Langlois and Rivlin [3]).

Stone [4] examined more closely the conditions under which the rectilinear motion of a non-Newtonian fluid or plastic solid is possible in tubes of non-circular cross-section. Truesdell and Noll [5] extended these previous analyses concerning the non-existence of rectilinear flow in a non-circular tube to general simple fluids. However, more recently, Fosdick and Serrin [6] have derived a more precise theorem concerning the general non-existence of rectilinear flow in non-circular tubes. They proved that (subject to the independence condition and certain analyticity and monotonicity conditions on the material functions) if a simple fluid undergoes a steady rectilinear motion in a straight tube whose cross-section is a bounded and connected open set, the tube must be either circular or the annulus between two concentric circles.

![Fig. 1. Non-Newtonian secondary flow in a tube of rectangular cross-section](image)

The purpose of the present paper is to examine more closely the role that the normal stresses play in the development of secondary flows. The analogous problem for the turbulent flow of a Newtonian fluid was examined recently in Speziale [7]. It will be proven independent of any constitutive assumptions that secondary flows will not occur in tubes of non-circular cross-section unless the axial velocity gives rise to some non-zero difference in the transverse normal stresses. Although it has long been known that such secondary flows constitute a nonlinear normal stress effect (Pipkin and Rivlin [8]), to the best knowledge of the author, this result appears to be one of the most general necessary conditions for the development of secondary flows to be derived in the literature. This criterion will be utilized to examine the role that the structure of the time derivatives in non-Newtonian fluids of the rate type and differential type play in the generation of secondary flows. It will be demonstrated that by a slight modification in the structure of the convected time rates in such non-Newtonian