Contact Problems of a Rectangular Block  
on an Elastic Layer of Finite Thickness  

Part II: The Thick Layer

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With 4 Figures

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Summary — Zusammenfassung

Contact Problems of a Rectangular Block on an Elastic Layer of Finite Thickness.  
Part II: The Thick Layer. We consider a layer of finite thickness loaded in plane strain by a stamp with a straight horizontal base, which is smooth and rigid. The stamp is pressed vertically into the layer and is slightly rotated by an external moment load subsequently. Two cases are considered successively: the lower side of the layer is attached to a rigid base and, secondly, the lower side is allowed to slide without friction along the base. The paper is divided into two parts. The first part deals with an asymptotic investigation of a thin layer and the second one refers to a thick one.

1. Introduction

For a general statement of the type of problem we will deal with, we refer to the introduction of Part I of this paper. The mathematical methods to be applied in the following differ from those used previously. We now perturb the known solution for an elastic half space and in this way we obtain approximate solutions in the form of uniformly convergent series in the ratio \( \frac{c}{b} \). It appears that the marked differences the thin layer shows in Case A between the solutions for incompressible and compressible material, are absent here. Moreover, in the limit \( b \to \infty \) the solutions for the cases A and B are identical as is obvious from a physical point of view.
2. A Rectangular Block with a Vertical Force

We will use the same coordinates, notations and units as in Part I. Referring to Fig. 1 and (2.6) of that part we have the following fundamental integral equation

\[ v_0 = \frac{b}{\sqrt{2\pi}} \int_{-1}^{+1} p(\xi) \frac{S(x - \xi)}{d\xi}, \quad (|x| \leq 1), \quad (2.1) \]

where the kernel \( S(x) \) follows from (2.7), (2.8) and (2.9) of Part I for the cases A and B, respectively. The solution \( p(x) \) of (2.1) has to satisfy the condition

\[ p(x) \geq 0, \quad (|x| \leq 1). \quad (2.2) \]

On account of the logarithmic unboundedness of the displacement at infinity, occurring for \( b \to \infty \), the method of the dual integral equations does not work for this particular problem (c.f. [4]) and therefore we proceed as follows. Substituting

\[ S(x) = \frac{1}{b} K \left( \frac{x}{b} \right), \quad (2.3) \]

(2.1) is transformed into

\[ v_0 = \frac{1}{\sqrt{2\pi}} \int_{-1}^{+1} p(\xi) K \left( \frac{x - \xi}{b} \right) d\xi, \quad (|x| \leq 1). \quad (2.4) \]

For small values of \( \frac{x}{b} \) we may write

\[ K \left( \frac{x}{b} \right) = -\sqrt{\frac{2}{\pi}} \log \left| \frac{x}{b} \right| + \sqrt{\frac{2}{\pi}} \sum_{k=0}^{\infty} \beta_k \left( \frac{x}{b} \right)^{2k}, \quad (2.5) \]

where the series converges uniformly if \( \left| \frac{x}{b} \right| < 1 \). We have calculated the first four coefficients for the two cases (Table 1) for several values of Poisson’s ratio.

<table>
<thead>
<tr>
<th>Case A</th>
<th>( v = 0 )</th>
<th>( v = 0.2 )</th>
<th>( v = 0.3 )</th>
<th>( v = 0.4 )</th>
<th>( v = 0.5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_0 )</td>
<td>-0.372</td>
<td>+0.574</td>
<td>-0.174</td>
<td>-0.392</td>
<td>+0.0135</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>-0.441</td>
<td>+0.647</td>
<td>-0.212</td>
<td>-0.392</td>
<td>+0.068</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>-0.527</td>
<td>+0.716</td>
<td>-0.245</td>
<td>-0.392</td>
<td>+0.080</td>
</tr>
<tr>
<td>( \beta_3 )</td>
<td>-0.683</td>
<td>+0.828</td>
<td>-0.298</td>
<td>-0.392</td>
<td>+0.100</td>
</tr>
</tbody>
</table>

Case B

| \( v \leq \frac{1}{2} \) | -0.352 | +0.137 | -0.083 | +0.025 |