Marangoni-Effect Velocity Distribution
Due to Time-Oscillatory Temperature Gradients
in Zero-Gravity Environment

By
H. F. Bauer, Neubiberg, Federal Republic of Germany

With 5 Figures

(Received July 30, 1981; revised September 3, 1981)

Summary
An infinite at both ends closed cylindrical free surface liquid column is subjected
to various instationary axial temperature time-wise periodic fields, such as linear, parabolic,
sinusoidal and exponential temperature changes, which are imposed at the free liquid
surface.

The flow velocity distributions due to such a thermal Marangoni effect is analytically
presented and numerically evaluated for a linear and parabolic temperature distribution.
The system is in a zero-gravity environment.

Nomenclature

- $a$: radius of liquid column
- $I_n$: modified Bessel function of first kind and order $n$
- $J_n$: Bessel function of first kind and order $n$
- $\text{ber}_n, \text{bei}_n$: Kelvin functions
- $p$: liquid pressure
- $r, \varphi, z$: polar cylindrical coordinates
- $u, w$: radial and axial velocity resp.
- $T$: temperature
- $t$: time
- $\nu = \frac{\eta}{\rho}$: kinematic viscosity
- $\Omega$: circular frequency
- $\sigma$: surface tension
- $\tau$: tangential stress
- $\eta$: dynamic viscosity

1. Introduction
Through the availability of extended space flights and manned earth-orbiting
laboratories unique manufacturing processings become possible, experiments
which hardly could be performed on earth under gravity conditions. In particular
floating zone melting may be employed for the purification of high melting
materials and for crystal growth. The melt does not require any crucible since
it floats freely in space in a micro-gravity environment and is held by surface tension. Such floating zone experiments have been performed by Sky-lab flights [1], [2], where crystal growth was pioneered under micro-gravity condition in orbit. But other areas for the investigation of basic scientific and technical problems with floating liquid bridges offer additional applications. The liquid bridge may therefore serve as a basic element of studies under microgravity environment [3]. The advantage of such a floating zone between two endplates in the absence of the destabilizing gravity effects is the relatively large magnitude of the length to diameter ratio, which can be achieved under micro-gravity. In addition to that buoyancy driven flow is eliminated.

In crystal growth it is well known that flow disturbances in the process of solidification are of a detrimental effect upon the perfection of crystalline materials. Although the reduction of gravity to very small or even zero gravity would improve the perfection of such crystals considerably, there are other interfering effects which may become predominant or which may introduce unwanted disturbances. Due to the heating of the material, there will even under zero gravity condition be convection processes [4] such as Marangoni convection [5] caused by the dependence of the surface tension upon the temperature. A particular feature of such a Marangoni convection flow is its becoming unstable beyond a critical point [6], [7], where the resulting oscillatory flow is connected with temperature oscillations. Such flow disturbances lead also to undesirable impurity striations or irregular shapes of the crystal. Numerous investigations have developed theoretical models for floating zones [8]. Using the technique of Plateau [9] many useful simulation experiments [10] have also been performed. It was found that theory is in reasonable good agreement with the experimental data as far as the hydrodynamics of cylindrical liquid bridges are concerned. Brice [11], however, mentioned, that the occurrence of too curved menisci results in a faceting of the growth front of the crystal with macroscopic heterogeneities in the material. This suggests that the cylindrical zone is possibly the most beneficial interface shape for crystal growth. This zone therefore has become the most widely used one [12], [13], not only because of its simpler geometry, but also because of the application of it as mentioned above. To obtain a melt the material has to be subjected to heat, which may not be uniform, such that temperature gradients are present during the processes. Differences in temperature, however, cause local variations in surface tension thus creating a convective effect upon the liquid. This effect is called thermal Marangoni convection. The velocity distribution due to such temperature fields have to be known before the heat conduction and mass transport problem may be solved adequately.

In addition there may be time-oscillatory temperature-gradients inducing an oscillatory flow in the liquid bridge. Four basic cases of stationary temperature distribution have been treated analytically [14]. The velocity distribution for linear, quadratic, sinusoidal and exponential temperature distributions were obtained by solving the Navier-Stokes equations for an infinitely long, but at both ends closed-up liquid column, observing the boundary condition at the surface in such a manner, that the tangential stress is equal to the gradient of the surface tension, as caused by the local variation of the temperature. The