Inviscid Fluid in High Frequency Excitation Field

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Summary

The influence of high frequency excitations (HFE) on a fluid is investigated. The response to these excitations is decomposed into two parts: “slow” motion, which practically remains unchanged during the vanishingly small period \( \tau \), and “fast” motion whose value during this period is negligible in terms of displacements, but is essential in terms of the kinetic energy. After such a decomposition the “slow” and “fast” motions become nonlinearly coupled by the corresponding governing equations. This coupling leads to an “effective” potential energy which imparts some “elastic” properties to the fluid and stabilizes laminar flows.

Introduction

The influence of high frequency excitations (HFE) on mechanical systems with a relatively low number of degrees of freedom is well known [1]. It can be shown that under HFE the motion of a mechanical system can be decomposed in two parts: “slow” motion, which practically remains unchanged during the vanishingly small period \( \tau \) of HFE, and “fast” motion, whose mean value during this period of time is negligible in terms of displacements but is essential in terms of the kinetic energy.

After such a decomposition the “slow” and “fast” motions become nonlinearly coupled by the corresponding governing equations. This coupling leads to an additional, “effective” potential energy which changes the dynamical characteristics of the original mechanical system.

The best illustration of this phenomenon is the stabilization of an inverted pendulum by vertical HFE imparted to its support.

Theoretical and experimental results concerning the interactions between mechanical systems and HFE give motivation for generalization of this phenomenon on continua, and particularly, on fluids.

In practical terms, HFE can be imparted by electromechanical transducers which have been successfully demonstrated for electronic damping of large flexible structures.
1. Governing Equations

Starting with the Euler equation of motion

\[ \rho \left( \frac{\partial v}{\partial t} + v \nabla v \right) = -\nabla p + F \]

(1.1)

and the continuity equation:

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 \]

(1.2)

where \( v, \rho, p \) are the velocity, the density and the pressure, respectively, as functions of the position vector \( r \) and time \( t \), and \( F \) is the external force, it will be assumed that the motion of the fluid is characterized by the time scale \( \tau \) and the distance scale \( l \) upon which the changes of the fluid parameters are negligible.

Suppose now that the fluid is subjected to additional external excitations characterized by frequencies of the order of \( \omega \) where

\[ \omega \ll \frac{1}{\tau}. \]

(1.3)

Then it is reasonable to expect that the response can be decomposed in two parts:

\[ v = v_1 + v_2 \cos (k \cdot r - \omega t), \quad v_2 \cdot k = 0 \]

(1.4)

where \( k \) is the wave vector, and

\[ |k| \ll \frac{1}{l} \]

(1.5)

while the velocities \( v_1 \) and \( v_2 \) are characterized by the scales \( \tau, l \).

Obviously,

\[ \int_0^{2\pi/k} v_2 \cos (k \cdot r - \omega t) \, dx = 0, \quad \frac{1}{l} \int_0^l v_i \, dx \simeq v_i, \quad (i = 1, 2), \quad x = \frac{k \cdot r}{k}. \]

(1.6)

The same decomposition (1.4) as well as the equalities (1.6) can be applied to \( p, \rho, \) and \( F \).

Substituting the decompositions into Eqs. (1.1) and (1.2) and integrating them over the interval \([0, 2\pi/k]\) in the direction of the wave vector \( k \), one ob-