Foundations of Extensional Viscometry

Part II: Essentials as to Design and Analysis of Hose Viscometers

By

M. Biemann, Berlin

With 3 Figures

(Received December 16, 1969)

Summary — Zusammenfassung

Foundations of Extensional Viscometry. Part II: Essentials as to Design and Analysis of Hose Viscometers. After a historical survey on the evolution of extensional viscometry, two novel types of extensional viscometers are proposed under the designations closed-end and open-end hose viscometer. The hose, which forms an elastically extensible cover of a prolate circular cylinder shape, is to hold an incompressible fluid specimen for investigation, especially under forced oscillations. The measurement of the first type concerns the axial force of the specimen in a thin-walled hose; the measurement of the second type concerns the radial traction of the specimen in a thick-walled hose. Fundamental design features and a mathematical treatment under ideal conditions are set forth for eventually showing how to unroll an informative (though expensive) program of viscometry on the base of harmonic Fourier analysis. In an appendix, the ascertainment of material coefficients is exemplified on specific constitutive assumptions so as to suggest analogously handling other instances on demand.

1. Historical and Introductory Remarks

Several workers have already recognized and emphasized the import of extensional viscometry as being instrumental to explore, especially, the nonlinear response of materials. The flows occurring in practical appliances are far from
being always simple shearing, the more so between convergent or divergent walls or near free surfaces. Thus it is significant to know the behavior under irrotational flows. But little progress in fundamental investigations of this kind has been notified as yet.

An elementary calculation under the assumption of linearly viscous incompressible (Navier-Stokes) fluids shows that the extensional flow resistance (defined as the quotient of the stress and the extension rate) equals thrice the shear-viscosity coefficient and is sometimes called Trouton viscosity. Trouton [1] was, as it seems, the first to employ extensional motions to the experimental study of very viscous materials such as pitch, whose viscosity ($10^5$ to $10^9$ Ns m$^{-2}$) is so great that the specimen flows only very slowly under its own weight. Similar experiments were performed on glass fibers with a viscosity even higher (up to about $10^{14}$ Ns m$^{-2}$) [2], [3].

Later on, Reiner [4] calculated the extensional flows of special nonlinear compressible materials, which are nowadays reckoned among the class of Reiner-Rivlin fluids, but did not check the compatibility of his assumptions with the basic equations of mechanics for the case of compressibility. Yet he noticed that the extensional flow resistance in linearly viscous materials of nonzero bulk-viscosity amounts to less than the Trouton viscosity as long as the dilatation is lasting. After the dilatation has decayed, the flow resistance of a Reiner-Rivlin fluid may, due to nonlinear effects, be larger or smaller than the Trouton viscosity and also depend on the sign of the extension rate in the sense that the resistance towards positive extension rates is, in case of a positive cross-viscosity, larger and, in case of a negative one, smaller than the resistance towards negative extension rates.

The investigations to be mentioned further were confined to incompressible fluids. Coleman and Noll [5] treated steady extensional motions of simple fluids (as defined by Noll) under consideration of inertia and found two material functions different from and, in general, not related to the so-called viscometric (in the sense of shearing) material functions. For fluids of the integral type, White [6] found that, already in the first order (which designates the number of integrations with respect to time in the material response functional), there appears an extensional flow resistance increasing with the extension rate — in accordance with the theory of rubberlike fluids established by Lodge [7] — although the shear-viscosity is constant. Bird and Stribeck [8] worked under the constitutive assumption of Oldroyd’s three-constant model — i.e. incompressible rate-type fluids of the first order (which designates the number of differentiations of stress and strain with respect to time) — and of similar models introduced by Stribeck with more constants. All these authors stated that the extensional

---

1 Their significance is corroborated also from a theoretical point of view since Noll [19] has shown that the response of so-called simple materials is fully knowable if and only if the response to all histories of homogeneous irrotational deformations is specified. Only present rotations are able to affect the present stress of simple materials. This theorem may, instead, figure as a definition for such materials and therefore, be of practical use merely if one knows from the outset that the material under consideration is simple in Noll’s sense. This supposition is also implied in the constitutive relations subsequently employed.