A variable-domain variational formulation of inverse problem $I_A$ of 2-D unsteady transonic flow around oscillating airfoils

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Summary. The present paper carries out, for the first time, a detailed theoretical investigation on the inverse problem in unsteady aerodynamics. Special attention is paid to finding proper ways of problem-posing and mathematical formulation. To demonstrate the basic idea, only an inverse problem of type $I_A$ of unsteady transonic flow with shocks around oscillating airfoils is studied herein. It has been formulated by a family of variational principles (VP) with variable domain, in which all unknown boundary (airfoil contour) and discontinuities (shocks and free trailing vortex sheets) are handled (captured) via the functional variation with variable domain. As a result, almost all boundary- and interface-conditions have been converted into natural ones. Thus, a rigorous theoretical basis for unsteady airfoil design and finite element (FE) applications is provided. On the basis of these variational principles developed in this paper, a method using new self-deforming finite element is suggested for the numerical realization of the variable-domain variation of the functional and a numerical example is given. Its suitability and effectiveness are demonstrated by the numerical results.

Notation

$A_1 \ldots A_4$ solution domain boundaries (Fig. 1)
$A_0^T$ solution domain in $R^3$ at $t = 0$ and $T$
$A_{fs}, A_{sh}$ free trailing vortex sheets and shock resp.
$i^*$ stagnation enthalpy
$n$ isentropic index
$R^2, R^3$ 2-D space $xy$ and 3-D space $xyt$ resp.
$\phi$ velocity potential
$\bar{A}, \bar{q}, \bar{p}$ fluid velocity, density and pressure resp.
$P_m$ time-averaged pressure nondimensionalized by a reference pressure, see the definition formulae just under Eq. (14)
$t, T$ time and period resp.
$x, y$ cartesian coordinates
$\xi, \eta, \zeta$ coordinates in image space
$V$ solution domain in $R^3$
$\delta$ variation symbol
$\llbracket X \rrbracket = X_+ - X_-$ is the jump in $X$ across discontinuity surface

Subscripts

$n, n'$ normal component in $R^2$ and $R^3$ resp.
$pr$ prescribed
$-, +$ the left and right sides of a discontinuity resp.
The overwhelming majority of the existing fluid mechanics literature is dedicated to the direct (analysis) problem, though the inverse problem is more directly oriented to and more useful for aerodynamic design. Only in the past decade, with the rapid development of computational fluid dynamics, more and more interest of scientists and engineers has been attracted to the inverse problem of steady aerodynamics [1]–[3].

As far as the authors are aware, however, up to now, there has not appeared any paper that deals with the inverse problem of unsteady flow.

It is our ambition to carry out, for the first time, a detailed theoretical investigation on the unsteady inverse problem. Special attention is paid to finding proper ways of problem-posing and mathematical formulation. First of all, in contrast to the steady flow case, in the unsteady inverse problem the pressure distribution along the airfoil contour can not be specified over the whole oscillation period in order to keep the airfoil geometry unchanged with time, while allowing the airfoil to move as a rigid body. Second, the unsteady inverse problem accommodates several different types, depending on different ways of specifying pressure distributions. In the present paper our attention will be concentrated on one type (designated by $I_A$) of the unsteady inverse problem: the time-averaged pressure distribution over the airfoil contour is given, while the corresponding airfoil shape is to be sought. For this $I_A$-problem, families of variational principles (VP) will be established on the basis of the authors’ previous papers dealing with the direct problem of unsteady flow [3]–[6].

On the basis of these variational principles, the finite element method (FEM) is employed for numerical solution of the inverse problem of 2-D unsteady transonic flow around oscillating airfoils, incorporating the nonreflecting far-field boundary conditions and a new unsteady Kutta condition [7]. All unknown boundary (airfoil contour) and discontinuities (shocks and free trailing vortex sheets) are handled (captured) via the functional variation with variable domain and artificial density concept. For the numerical realization of the variable-domain variation, a special finite element with self-adjusting nodes is also suggested herein [17].

2 Basic equations of 2-D unsteady transonic airfoil flow

The dimensionless basic aerodynamic equations have the following form:

\[ \frac{\partial \phi}{\partial t} + \nabla \cdot (\rho \vec{A}) = 0, \]

\[ \nabla \phi = \vec{A}, \]