Note

Thermal boundary layer flow of a micropolar fluid past a wedge with constant wall temperature

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Summary. The steady laminar flow of micropolar fluids past a wedge has been examined with constant surface temperature. The similarity variables found by Falkner and Skan are employed to reduce the streamwise-dependence in the coupled nonlinear boundary layer equation. Numerical solutions are presented for the heat transfer characteristics with Pr = 1 using the fourth-order Runge-Kutta method, and their dependence on the material parameters is discussed. The distributions of dimensionless temperature and Nusselt number across the boundary layer are compared with the corresponding flow problems for a Newtonian fluid over wedges. Numerical results show that for a constant wedge angle with a given Prandtl number Pr = 1, the effect of increasing values of K results in an increasing thermal boundary thickness for a micropolar fluid, as compared with a Newtonian fluid. For the case of the constant material parameter K, however, the heat transfer rate for a micropolar fluid is lower than that of a Newtonian fluid.

Nomenclature

- $h$: Dimensionless microrotation
- $j$: Micro-inertia density
- $K$: Dimensionless parameter of vortex viscosity
- $m$: Falkner-Skan power-law parameter
- $Re$: Reynolds number
- $T$: Temperature
- $u,v$: Fluid velocities in the $x$ and $y$ directions, respectively
- $U$: Free stream velocity
- $x$: Streamwise coordinate along the body surface
- $y$: Coordinate normal to the body surface

Greek symbols

- $\alpha$: Thermal diffusivity
- $\beta$: Wedge angle parameter
- $\gamma$: Spin gradient viscosity
- $\eta$: Pseudo-similarity variable
- $\kappa$: Vortex viscosity
- $\mu$: Absolute viscosity of the fluid
- $\nu$: Kinematic viscosity
1 Introduction

The theory of micropolar fluids which displays the microscopic effects arising from the local structure and micro-motions of the fluid elements was formulated by Eringen [1], [2]. This is a recent development in continuum mechanics, and has attracted the attention of several investigators (Bergholz [3], Emra and Kulacki [4], Chandra Shekar et al. [5]) because of their industrial applications. Physically, the mathematical model underlying the micropolar fluid may represent the behavior of polymeric additives, blood, lubricants, liquid crystals, dirty oils and colloidal suspension solutions.

The theory of such fluids has been applied to a wide range of classical flows. For example, Rees and Bassom [6] have studied the micropolar analogue of the Blasius boundary layer flow. They derived nonsimilar boundary layer equations and solved them using the Keller-box method. They also performed an asymptotic analysis for large distances from the leading edge because the numerical results indicate that the boundary layer develops a two-layer structure.

Gorla [7] applied the micropolar boundary layer theory to the problem of two-dimensional steady stagnation point flow with constant wall temperature. He mentioned that the numerical results could be used for drag reduction purposes or heat transfer rate augmentation in heat exchangers. Gorla [8], [9] also investigated the boundary layer characteristics of an axisymmetric, laminar, micropolar fluid flow with a uniform velocity \( U \) along a horizontal cylinder. He presented numerical solutions for the velocity, micro-rotation and heat transfer fields for a wide range of values of the dimensionless curvature parameter as well as material parameters.

In Sect. 2 we summarize the equations governing the characteristic micropolar flow fields over the wedge with the appropriate boundary conditions. In subsequent sections detailed numerical results are presented for the heat transfer characteristics, and their dependence on material parameters of the fluid is discussed.

2 Formulation of problems

Consider a two-dimensional steady flow of a laminar, incompressible, micropolar fluid flow past a wedge with constant wall temperature. The physical model and geometrical coordinates are shown in Fig. 1. The effects of viscous dissipation will be assumed to be negligible. Under the usual boundary layer approximation, the governing equations for the steady, laminar, incompressible, micropolar fluid flow past a wedge can be casted into nondimensional form with the absence of body forces and body couples, as follows:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{1}
\]