Wave Propagation and Reflection in Liquid Filled Distensible Tube Systems Exhibiting Dissipation and Dispersion

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With 8 Figures

(Received March 18, 1985; revised June 3, 1985)

Summary

In a previous paper the present authors developed a model describing wave propagation in liquid filled distensible tubes and tested it against impulse experiments involving water filled latex rubber tubes. This model incorporates both dissipative and dispersive mechanism which are absent from the commonly employed linear long wavelength (LLW) theory of haemodynamics. This higher order theory is here employed to study propagation of an impulse in a semi infinite tube, reflection of an impulse from the distal end of a finite length tube, and reflection and transmission of impulses impinging on a junction connecting dissimilar liquid filled tubes. Both open and closed type reflections are treated and numerical results presented graphically. To the best of our knowledge this is the first time that such a higher order theory has been employed to treat reflection and transmission of waves in tube systems.

1. Introduction

Theoretical studies which involve modelling of wave propagation in liquid filled distensible tubes very often have as their motivation the desire to understand aspects of the cardiovascular system in both its normal and pathological states. This is no less true for experimental studies in this area ([1]—[5]).

The theory of pressure wave propagation that appears most often in the haemodynamics literature is a linear long wavelength (LLW) theory based upon ideas first put forward by Young [6]. Lighthill [7] and others have refined and extended these ideas to apply in a variety of situations representative of the cardiovascular system. This LLW theory, when applied to reflections arising from abrupt area/elasticity changes, has been shown to give good agreement with experiment (see [1]—[4]). The LLW theory gives good agreement with these reflection experiments precisely because the changes (geometrical/mechanical)
take place over a length $L$ which is very short in comparison to a typical wavelength $\lambda$ (that is, $L/\lambda < 1$). This theory, however, usually fails to model adequately the transmission characteristics of the pulse since in this instance the long wavelength condition is $2R/\lambda \ll 1$, where $R$ is the tube radius [8] and this is not usually satisfied in practice. For example, in the experiments of [3] and [4], $2R/\lambda \simeq 0.2$, and the long wavelength condition is violated. The kinematical manifestation of this violation may be observed in Fig. 1 of [3] where it is seen that the propagating pulse undergoes a pronounced change in shape with transmission. The pulse broadens, is attenuated, and its oscillatory tail is damped as the wave travels down the tube.

In a previous paper [9] the present authors developed and tested a simple linear theory for the transmission of pressure pulses in liquid filled distensible tubes which overcomes the deficiencies in the LLW theory. It is the purpose of this paper to employ this theory to demonstrate the effect of mechanical mismatching on reflections of pressure waves. This study has application to the early detection of sites of arterial disease. In previous studies on the arterial system it is the geometrical discontinuities and, in particular, the narrowing or stenosis of a vessel which have received the most attention. This was partly because atherosclerotic narrowing is a common and obvious feature of cardiovascular disease, but also because the possible generation of turbulence, in a partially obstructed blood vessel, was an attractive problem for fluid dynamicists. In contrast, mechanical mismatching has received little or no attention. Yet wall property changes and hence a local change in wave speed is a common feature of atherosclerosis and occurs prior to arterial narrowing. In addition, surgical bypass techniques involve the use of autogenous or prosthetic materials, which, although they are geometrically matched, may differ greatly from normal arteries in their mechanical properties.

2. The Model

Since the detailed development of the mathematical model to be employed here has appeared elsewhere [9] we shall simply summarize the equations and define the various quantities involved.

The equation of motion for the tube wall is

$$2(\tau_c D + 1) W = P - \frac{\gamma \alpha}{\rho} \frac{\partial^2 W}{\partial t^2},$$

where $\tau_c$ is a characteristic time for the tube wall material, $D = \frac{\partial}{\partial t}$, $W$ is the radial wall displacement, $P$ is the pressure difference across the tube wall, $\gamma$ is the density of the tube wall material, $\rho$ is the density of the fluid and $\alpha = h/R$ where $h$ is the tube wall thickness and $R$ the tube radius. Throughout this paper non-