Global Stabilization of a Dynamic von Kármán Plate with Nonlinear Boundary Feedback*

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Abstract. We consider a fully nonlinear von Kármán system with, in addition to the nonlinearity which appears in the equation, nonlinear feedback controls acting through the boundary as moments and torques. Under the assumptions that the nonlinear controls are continuous, monotone, and satisfy appropriate growth conditions (however, no growth conditions are imposed at the origin), uniform decay rates for the solution are established. In this fully nonlinear case, we do not have, in general, smooth solutions even if the initial data are assumed to be very regular. However, rigorous derivation of the estimates needed to solve the stabilization problem requires a certain amount of regularity of the solutions which is not guaranteed. To deal with this problem, we introduce a regularization/approximation procedure which leads to an "approximating" problem for which partial differential equation calculus can be rigorously justified. Passage to the limit on the approximation reconstructs the estimates needed for the original nonlinear problem.

Key Words. Stabilization, Nonlinear boundary feedback, von Kármán plate, Robustness.

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1. Introduction

1.1. Statement of the Problem

Let \( \Omega \) be an open bounded domain in \( \mathbb{R}^2 \) with a sufficiently smooth boundary, \( \Gamma \). In \( \Omega \) we consider the following von Kármán system in the variables \( w(t, x) \) and \( \chi(w(t, x)) \) with nonlinear feedback controls, \( g \), \( f_1 \), and \( f_2 \):

\[
\begin{align*}
\varepsilon \Delta w_{tt} + \Delta^2 w + b(x)w_t &= [w, \chi(w)] \\
&= g(w_t) - f_1 \left( \frac{\partial}{\partial v} w_t \right) \\
&\text{in } \Omega \times (0, \infty), \quad (1.1a) \\
\Delta w + (1 - \mu)B_1 w &= -f_1 \left( \frac{\partial}{\partial v} w_t \right) \\
&= g(w_t) - f_2 \left( \frac{\partial}{\partial \tau} w_t \right) \\
&\text{on } \Sigma_\infty = (0, \infty) \times \Gamma, \quad (1.1b) \\
\Delta w + (1 - \mu)B_2 w - \gamma^2 \frac{\partial}{\partial v} w_{tt} &= -fa \quad \text{in } \Omega \times (0, \infty), \quad (1.1c) \\
\Delta w - (1 - \mu)B_2 w - \gamma^2 \frac{\partial}{\partial \tau} w_{tt} &= -f_2 \quad \text{on } \Sigma_\infty = (0, \infty) \times \Gamma, \quad (1.1d)
\end{align*}
\]

where \( b(x) \in L^\infty(\Omega) \) satisfies \( b(x) > 0 \) a.e. \( \Omega \), \( 0 < \mu < \frac{1}{2} \) is Poisson's ratio, the operators \( B_1 \) and \( B_2 \) are given by

\[
\begin{align*}
B_1 w &= 2n_1 n_2 w_{xy} - n_1^2 w_{yy} - n_2^2 w_{xx}, \\
B_2 w &= \frac{\partial}{\partial \tau} \left[ (n_1^2 - n_2^2)w_{xy} + n_1 n_2 (w_{yy} - w_{xx}) \right].
\end{align*}
\]

with \( v = (n_1, n_2) \) outward normal to the boundary, \( \tau \) is tangential. The controls \( g \) and \( f_i \) are continuous, monotone functions and are subject to the following constraints:

\[
\left\{ \begin{array}{l}
g(s)s > 0 \quad \text{for } s \neq 0, \\
f_i(s)s > 0 \quad \text{for } s \neq 0, \\
m|s| \leq |f_i(s)| \leq M|s| \quad \text{for } |s| > 1, \quad i = 1, 2, \\
g(s)s \leq M|s|^{r+1} \quad \text{for } |s| > 1,
\end{array} \right.
\]

where \( r \) is any positive constant. The parameter, \( \gamma \), in (1.1a) is proportional to the thickness of the plate and is therefore assumed to be small.

Remark 1.1. No assumptions are made on the behavior of \( g \) and \( f_i, i = 1, 2 \), at the origin.