SENSITIVITY AND ROBUST STABILITY OF NONLINEAR INPUT-OUTPUT SYSTEMS CONSISTING OF FINITELY MANY BLOCKS*

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Abstract. We construct a general model of a nonlinear input-output system consisting of finitely many blocks whose variables are in extended spaces. This model encompasses all configurations used in control which have several feedback and feedforward loops. We show that such a system is essentially the traditional MI-MO feedback system. Moreover, assuming that one block of the system is a plant affected by perturbations, we derive conditions guaranteeing linear insensitivity and/or robust stability of the whole system. In particular, we consider systems whose variables are continuous, vector-valued functions on $[0, \infty)$, and whose blocks are described by nonlinear Volterra operators. The applications of the results are illustrated by several examples of feedback-feedforward systems.

1. Introduction

The purpose of this paper, which appears as a certain follow-up to [3], is to construct a general model of a nonlinear input-output system that consists of finitely many interconnected blocks. Such systems are encountered in the design of modern control systems [1], power amplifiers [5], VLS systems, and the like. In particular, our model is to encompass systems having several feedback and feedforward loops.

To this end, we assume that such an input-output system, which will be called a "block system" in the sequel, is specified by (i) transmission operators $B_i$, $i = 1, 2, \ldots, n$, of individual blocks, (which are not necessarily linear), (ii) a constant matrix $C$ that describes the interconnection of outputs of individual blocks with inputs of other blocks, (iii) a constant matrix $d$ that describes how an input $z$ of the whole system is fed to inputs of individual blocks, and (iv) a constant matrix $a$ that determines how the output $y$ of the whole block system is derived from the outputs of blocks. To ensure that the applicability of the model is as wide as possible, it is

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assumed that the variables of the block system are in general extended spaces [2, Part I].

This definition of a block system is given in Section 2 of the paper. It turns out that, in this interpretation, a block system is governed by three simple operator equations (6)-(8). Based on these equations, it is shown that any block system is in fact a traditional MI-MO feedback system preceded by an input-output system described by the matrix $d$, and followed by an input-output system described by the matrix $a$.

The main topic of the paper — the sensitivity and robust stability — is discussed in Section 3. It is assumed that one block of the block system, say block $B_1$, is a plant affected by perturbations. Based on equations (6)-(8) it is shown that our block system $S(A)$, where $A$ is the transmission operator of the perturbed plant, can be identified with an abstract input-output system $[T, P]$, which was studied in [2, Parts I and II] and [4]. It turns out that two different identifications are possible, the first one being given by (22), and the second one by (24).

Using then the general theorems on the abstract input-output systems given in [2, Part I] and [4], we establish several conditions guaranteeing linear insensitivity of the nominal system $S(A_0)$ with respect to the set of bounded perturbations $A - A_0$ of the nominal plant $A_0$, and conditions for robust stability on the set of perturbations that are Lipschitz continuous (Theorems 1 and 2).

Finally, Theorem 3 deals with the most frequent case, a block system whose variables are continuous, vector-valued functions on $[0, \infty)$, and whose blocks are described by nonlinear Volterra operators $B_i$, $i = 1, 2, \ldots, n$. It is shown that the operators $B_i$ along with the matrix $C$ determine the underlying extended space $C_{-\infty}$, with respect to which the block system exhibits linear insensitivity and robust stability. (Note that $C_{-\infty}$ is the extended space of continuous functions on $[0, \infty)$ whose core $C_{-\infty}$ consists of continuous functions on $[0, \infty)$ that are bounded by multiples of the exponential $e^{\lambda t}$, see (36).) The validity of the assumptions of Theorem 3 can be easily tested in specific cases of block systems, in particular when the blocks $B_i$ are built from constant elements.

The presented theory is illustrated by several examples of specific block systems.

2. System description.

The concepts and the terminology we will use are the same as in [2]. In particular, the extended space $\tilde{H}$ with core $H$ and projections $P_a : \tilde{H} \to H$, $a \in \mathcal{T}$ is defined in [2, Part I, p. 365]. On the same page, homochronicity of extended spaces is introduced. If $\tilde{H}$, $\tilde{K}$ are homochronous spaces, the symbol $\mathcal{N}(\tilde{H}, \tilde{K})$ denotes the linear space of all (not necessarily linear) operators $N : \tilde{H} \to \tilde{K}$.

The class $\text{Lip}^*(\tilde{H}, \tilde{K})$ of Lipschitz operators in $\mathcal{N}(\tilde{H}, \tilde{K})$ (with seminorm $\| \cdot \|$) is defined in [2, Part I, p. 366], and the class $\mathcal{B}_0^*(\tilde{H}, \tilde{K})$ of bounded operators in $\mathcal{N}(\tilde{H}, \tilde{K})$ (with norm $\| \cdot \|_0$) is introduced in [2, Part II, p. 449].