A Multi-Continuum Theory for Composite Elastic Materials

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With 2 Figures

(Received August 21, 1970)

Summary — Zusammenfassung

A Multi-Continuum Theory for Composite Elastic Materials. A continuum theory for composite materials is presented in which the composite constituents are modeled by superimposed continua which undergo thermal and mechanical interactions. Kinematical notions, field equations and a constitutive theory are developed, including consequences of material frame indifference and material symmetry which restrict the form of the response functions. The final result is a set of linearized equations for thermal and mechanical processes in laminated or fiber reinforced composite elastic materials.


1. Introduction

The increasing application of a broad variety of composite materials has led to a need for analytical techniques for solving mechanical and thermal problems involving these materials. The characteristic approach to such problems in the past has been to model the composite material by a single continuum, occasionally anisotropic, whose constitutive constants are chosen in such a way that the behavior of the single continuum provides an approximation to the behavior of the composite [1], [2], [3], [4]. Although this approach has been found adequate for static and quasi-static problems, it loses completely any characterization of the macroscopic structure of the material. It is well known that structural effects give rise to dispersion and dissipation in dynamical processes in composite materials which the usual single continuum theories are intrinsically incapable of exhibiting [5], [6].

In this paper, we present a thermomechanical theory for composite materials in which the composite constituents are modeled by individual superimposed continua which may interact thermally and mechanically. In contrast to previous theories intended for application to bonded composite materials, we permit each
constituent to undergo an individual motion. The mechanical interactions between the individual constituent motions then provide a means of including composite structural effects in the theory.

Our concept can be motivated intuitively using figure 1, which illustrates a typical displacement profile resulting from the propagation of a stress wave in a fiber reinforced or laminated composite material. As a result of the different elastic properties of the two constituents, the average displacement in the two constituents can differ. This difference in average displacement will give rise to a strong mechanical interaction at the interface between the constituents. To model processes of this type using superimposed continua, we interpret the displacements in the individual continua to be equivalent to the average displacements in the composite constituents. Therefore, since the average displacements in the constituents may differ, we permit the superimposed continua to move relative to one another. In order to account for the interactions between the constituents at the interfaces, we introduce mechanical interactions between the continua which depend on the constituent relative displacements.

In reference [7], a purely mechanical, one dimensional theory of this type was postulated and used to successfully predict the dispersion and attenuation of steady state plane waves in laminated composite materials. We now develop a general nonlinear thermomechanical theory of interacting continua in a context applicable to bonded composite materials. In order to incorporate relative displacement coupling into the theory, we introduce the concept of closely coupled interacting continua in the next section and briefly develop essential kinematical notions. Field equations appropriate for closely coupled continua are exhibited in section 3, and the nonlinear constitutive theory is presented in section 4. In the final section, we linearize the general theory to obtain equations for thermal and mechanical processes in fiber reinforced and laminated composite materials. In their one dimensional form, the mechanical equations obtained reduce to the equations used in [7].

We use Cartesian tensor notation in the main development, with lower case latin indices referring to a spatial coordinate system and having a range of 1, 2, 3. Indices enclosed in parentheses differentiate between constituents and the sum-