On the stress wave induced curving of fast running cracks – a numerical study by a time-domain boundary element method

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Summary. Fast crack propagation in dynamically loaded plane structures is investigated. The major point of interest is the evolution of the crack trajectory under the influence of stress waves which are generated and repeatedly reflected at the specimen boundaries. Since these waves may lead to arbitrary mixed-mode and time-dependent loading of the crack tip, both the direction and speed of crack advance are determined from a fracture criterion.

Starting point is a system of time-domain boundary integral equations which describes the initial boundary value problem of a linear elastic body containing an arbitrarily growing crack. The unknown displacements and/or tractions on the exterior boundary and the displacement jumps across the crack are computed numerically by a collocation method in conjunction with a time-stepping scheme. Crack growth is modelled by adding new boundary elements of constant length at the running crack tip.

The method proves to be of sufficient accuracy when applied to problems treated with other numerical techniques. Moreover, the simulation of dynamic crack propagation under various geometry and loading conditions enables the reproduction and analysis of complex phenomena observed experimentally.

1 Introduction

Material inertia may affect crack propagation considerably. Inertia effects can arise either from the rapid motion of the material in the vicinity of a fast running crack tip or from stress waves travelling through a cracked body. While cracks observed in reality hardly reach a speed at which the dynamic crack tip stress field differs qualitatively from the static one [8], the influence of stress waves on a fracture process can often be observed in experiments or in real structures, especially in brittle materials. Stress waves – e.g. produced by a rapidly applied loading on the boundary of a body – transmit the information about geometry and loading conditions to the crack. Because of the finite wave speeds this information reaches the crack from different parts of the boundary at different times – in contrast to quasistatic loading. Furthermore, stress waves are reflected at the boundaries and thus may repeatedly interact with the running crack.

For example, experiments performed by Knauss and Ravi-Chandar [10] with specimens of different size but identical loading conditions showed a stronger effect of stress waves on the crack path in the smaller specimens due to the higher number of reflections. A characteristic S-shape is shown by the crack path in a bend specimen under impact loading when the crack
approaches the compression side of the specimen [13]. The oscillating crack path often observed in pipes under internal pressure results from stress waves propagating in circumferential direction in connection with a tensile stress in crack-parallel direction causing the crack path instability [10].

Obviously, in a realistic model for the theoretical investigation of the aforementioned phenomena the temporal and spatial evolution of a crack must not be prescribed. They have to be controlled only by a physically meaningful fracture criterion. For the solution of the respective initial boundary value problem that means that the crack trajectory itself has to be determined from the analysis, which in general can only be accomplished by numerical methods.

In recent years different numerical techniques have been employed for the simulation of 'free' dynamic crack propagation under mixed-mode loading. Swenson and Ingraffea [16] applied a finite element method where the data transfer on the mesh which partly moves with the crack tip causes additional numerical expense. The element-free Galerkin method developed by Belytschko and Tabbara [2] is capable of modelling arbitrary crack paths without mesh distortion but is also numerically expensive and has only been applied to crack growth at constant speed. The direction of crack advance in both approaches is taken to be that of maximum circumferential stress. Xu and Needleman [17] extended a finite element method by spatially distributed cohesive surfaces which allows to simulate arbitrary crack growth at variable speed (including crack branching) without any fracture criterion. But, due to the enormous number of degrees of freedom the cohesive surfaces approach seems to be restricted to small specimen dimensions and time ranges.

An alternative to these methods of domain discretization are boundary element methods which are based on a total problem reduction to the boundary including the crack. Therefore, they are well suited for modelling curvilinear crack propagation in linear elastic (brittle) materials. Koller et al. [11] investigated the unsteady propagation of a longitudinal shear crack (mode III) in an unbounded domain up to the formation of a first kink. Crack growth was modelled simply by adding new elements of a constant length to the moving crack tip which means that no remeshing was required. An analogous approximation was applied by Seelig and Gross [15] to dynamic crack propagation under plane strain conditions in an unbounded domain. For arbitrary mixed-mode loading – also taking into account the effect of crack face contact – curvilinear crack growth at variable speed was simulated. The direction and rate of crack growth were determined from the fracture criterion of maximum circumferential stress. In the range of realistic crack tip speeds ($\dot{a} < 0.6c_T$) the computed trajectories of cracks under external static or step stress wave loading did not differ much from those obtained from fully static analyses since in an unbounded domain scattered waves are radiated to infinity. Dynamic crack propagation at constant speed in a finite plate was investigated by Fedelinski et al. [7] using a ‘dual boundary element method’ and a modelling of crack growth similar to that in [11] and [15].

In the present paper the method applied in [15] is extended to dynamic crack propagation in finite structures with the aim to analyse the influence of multiply reflected stress waves on a fracture process. First, a system of time-domain boundary integral equations governing the respective initial boundary value problem and a suitable fracture criterion are presented (Sec. 2). Then the procedure for the numerical solution of the integral equations, the discrete modelling of crack growth, and consequences of this discretization to the appropriate evaluation of the fracture criterion are discussed (Sec. 3). Numerical results are shown in Sec. 4 for fracture processes in plates under various loading conditions. The influence of several parameters on typical phenomena like those mentioned at the beginning is analysed.