On certain Transformations of nearly-poised
Basic bilateral Hypergeometric Series of the Type $M \Psi_M$

By

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1. Introduction. Slater [4] in 1952 gave relations involving general and well-poised $M \psi_M$ series, but relations connecting nearly-poised series of the type $M \psi_M$ were not considered by her. In this paper I consider Slater's transformation to obtain certain relations involving nearly-poised bilateral hypergeometric series of the type $M \psi_M$. The main interest in considering these relations is in giving particular values to $M$ to obtain special transformations of nearly-poised series with the help of known transformations.

The following notation is used throughout the paper:

\[ (a; n) = (1 - a) (1 - a q) \cdots (1 - a q^{n-1}); \quad (a; 0) = 1 \]
\[ (a; n) = a (a + 1) \cdots (a + n - 1); \quad (a)_0 = 1; \quad (a)_{-n} = (-1)^n/(1 - a)_n \]
\[ (a; b)_n = \frac{(a)_n (b)_n}{(a + b)_n}; \quad (a; b)_0 = 1; \quad (a; b)_{-n} = (-1)^n/(1 - a - b)_n \]
\[ a_1, a_2, \ldots, a_r, \ldots, a_M; b_1, b_2, \ldots, b_M \]
\[ (a; b, c)_n = \frac{(a)_n (b)_n (c)_n}{(a + b + c)_n}; \quad (a; b, c)_{-n} = (-1)^n/(1 - a - b - c)_n \]

and idem $(a; b)$ means that the preceding expression is repeated with $a$ and $b$ interchanged.

2. Slater [4] gave the following relation which expresses a general bilateral $M \psi_M(x)$ series in terms of $M$ other series of the same type:

\[ \begin{align*}
\Pi \left[ x A, q/x A, b_1, a_2, \ldots, b_M, q/c_1, q/c_2, \ldots, q/c_M; \right] & = \frac{q}{a_1} \Pi \left[ x A, q/x A, b_1, a_2, \ldots, b_M, q/c_1, q/c_2, \ldots, q/c_M; \right] \\
& \times \Psi_M \left[ c_1, c_2, \ldots, c_M; x \right] \\
& \psi_M \left[ a_1, a_2, \ldots, a_M, q/a_1, q/a_2, \ldots, q/a_M; \right] \\
& \times \psi_M \left[ b_1, b_2, \ldots, b_M; \right]
\end{align*} \]

where

\[ A = \frac{c_1 c_2 \cdots c_M}{a_1 a_2 \cdots a_M}, \quad |x| < 1, \quad \frac{b_1 b_2 \cdots b_M}{c_1 c_2 \cdots c_M x} < 1. \]
If we first put \(x = \frac{a_1 a_2 \ldots a_M}{q c_1 c_2 \ldots c_M}\) in (2.1) in order to make the series on the left vanish and then substitute \(a_1 = q, b_2 = c_1 b_1/c_2, b_3 = c_1 b_1/c_3, \ldots, b_{M-1} = c_1 b_1/c_{M-1}\), we get the following relation between \(M\) nearly-poised series of the type \(\Phi_M\):

\[
\Pi \left[ \frac{q}{a_1}, \frac{q}{a_3}, \ldots, \frac{q}{a_M}, \frac{q}{a_1 a_3 \ldots a_M} \right] \times \frac{q}{a_2 a_3 \ldots a_M, a_2 a_3 \ldots a_M} \times \Phi_M \left[ \frac{a_2 a_3 \ldots a_M}{c_1 c_2 \ldots c_M}, \frac{a_2 a_3 \ldots a_M}{c_1 c_2 \ldots c_M} \right]
\]

\[
= \Pi \left[ \frac{a_2 a_3 \ldots a_M}{c_1 c_2 \ldots c_M}, \frac{a_2 a_3 \ldots a_M}{c_1 c_2 \ldots c_M} \right] \times \Phi_M \left[ \frac{a_2 a_3 \ldots a_M}{c_1 c_2 \ldots c_M}, \frac{a_2 a_3 \ldots a_M}{c_1 c_2 \ldots c_M} \right] + \text{idem} (a_2, a_3, a_4, \ldots, a_M),
\]

where

\[
\left| a_2 a_3 \ldots a_M \right| < 1 \quad \text{and} \quad \left| \frac{a_2 a_3 \ldots a_M}{c_1 c_2 \ldots c_M} \right| < 1.
\]

**Particular cases:**

(i) If we put \(b_1 = q\) in (2.2), we get a relation between a nearly-poised \(\Phi_{M-1}\) series of the second kind and \((M - 1)\) nearly-poised series of the type \(\Psi_M\).

(ii) Again if we put \(b_M = a_M\) in (2.2), we get the following relation between \((M - 1)\) nearly-poised \(\Psi_M\) series and a nearly-poised \(\Phi_{M-1}\) series of the first kind:

\[
\Pi \left[ \frac{q}{a_1}, \frac{q}{a_3}, \ldots, \frac{q}{a_M}, \frac{q}{a_1 a_3 \ldots a_M} \right] \times \Phi_M \left[ \frac{a_2 a_3 \ldots a_M}{c_1 c_2 \ldots c_M}, \frac{a_2 a_3 \ldots a_M}{c_1 c_2 \ldots c_M} \right] \times \Phi_M \left[ \frac{a_2 a_3 \ldots a_M}{c_1 c_2 \ldots c_M}, \frac{a_2 a_3 \ldots a_M}{c_1 c_2 \ldots c_M} \right] + \text{idem} (a_2, a_3, a_4, \ldots, a_M) +
\]

\[
(2.3)\]