ON REAL AND COMPLEX NORMALIZATIONS*

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Abstract. This paper presents a general expression relating the complex-normalized scattering matrix of an n-port network to that of its augmented n-port network normalizing to the $n 1 - \Omega$ resistances, where the Darlington equivalent network may be either reciprocal or nonreciprocal.

1. Introduction

In the design of a broadband matching equalizer, an elegant real-frequency method was introduced by Carlin [1]. His method was extended to solve the double matching problem by introducing the real-normalized scattering matrices [2], and to the problem of reciprocal reactance 2n-port cascade decomposition by Youla, Carlin and Yarman [3]. A general solution covering both the reciprocal and nonreciprocal decompositions was obtained by Uruski [4]. Recently, an interesting interpretation regarding a relation between the complex normalization and the Darlington theorem was given by Pauli [5]. Although there are different complex normalizations, it is commonly believed [2]–[5] that the complex normalized scattering matrix equals the unit-normalized scattering matrix of the augmented network on a one-to-one basis.

Let $z(s)$ be an $n \times n$ diagonal non-Foster positive-real impedance matrix, the reciprocal Darlington representation of which is $\tilde{N}$. According to Wohlers' definition of complex normalization [6], as followed by Chien [7], Carlin and Yarman [2] and Chen [8], Zhu and Chen [9] showed that the scattering matrix of an n-port network $N$ normalizing to $z(s)$ is different from the unit-normalized scattering matrix of the $n$-port network $N_a$ composed of the cascade connection

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of the lossless \( n \)-port \( N \) and \( 2n \)-port \( \bar{N} \), as indicated in Figure 1, when the para-hermitian part
\[
\mathbf{r}(s) = \frac{1}{2} \left[ \mathbf{z}(s) + \mathbf{z}^T(-s) \right]
\] contains zeros in the right-half of the \( s \)-plane (RHS), where \( \mathbf{z}^T(s) \) denotes the transpose of \( \mathbf{z}(s) \).

In this paper, we review the concept of complex normalization, and then presents a general expression relating the complex-normalized scattering matrix of the \( n \)-port network \( N \) to its augmented \( n \)-port network \( \bar{N}_a \) normalizing to the \( n 1 - \Omega \) resistances, where the Darlington equivalent networks may either be reciprocal or nonreciprocal. In light of the present result, we clarify some of the earlier conclusions [3], [4].

2. Review of the complex normalization

Given \( n \) rational, non-Foster positive-real functions \( z_1(s), z_2(s), \ldots, z_n(s) \) and a lumped passive \( n \)-port \( N \), set
\[
r_k(s) = \frac{1}{2} [z_k(s) + z_k(-s)] = h_k(s)h_k(-s), \quad k = 1, 2, \ldots, n
\] (2)
\[
d_k(s) = h_k(s)h_k^{-1}(-s), \quad k = 1, 2, \ldots, n.
\] (3)

Youla [10] showed that if the decomposition of (2) satisfies the following three conditions:

1) \( h_k(s) \) is analytic in \( \text{Re} \, s > 0 \),