A Generalised Model of Milling Forces

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This paper presents the development of a generalised cutting force model for both end-milling and face-milling operations. The model specifies the interaction between workpiece and multiple cutter flutes by the convolution of cutting-edge geometry function with a train of impulses having the period equivalent to tooth spacing. Meanwhile, the effect of radial and axial depths of cut are represented by the modulation of the cutting-edge geometry function with a rectangular window function. This formulation leads to the development of an expression of end-face-milling forces in explicit terms of material properties, tool geometry, cutting parameters and process configuration. The explicitness of the resulting model provides a unique alternative to other studies in the literature commonly based on numerical integrations. The closed-form nature of the cutting force expression can facilitate the planning, optimisation, monitoring, and control of milling operations with complicated tool-work interactions. Experiments were performed over various cutting conditions and results are presented, in verification of the model fidelity, in both the angle and frequency domains.

Keywords: Forces; Generalised; Milling; Model

1. Introduction

Cutting force is an important physical variable that embodies relevant process information of machining. Such information can be used to understand process attributes related to machinability, cutter wear/fracture, chatter, part dimensional accuracy and surface finish [1-3]. The modelling of milling forces with complicated tool and workpiece geometry generally involves the determination of local cutting forces at one cutting point and the integration of the local cutting forces along cutting edges. Koenigsberger and Sabberwal [4] treated the local tangential force as the product of a specific cutting pressure and instantaneous chip load. Tlusty and MacNeil [1] considered the radial local force to be proportional to the tangential local force. These basic relationships have been widely used by other researchers [2,3,5-8] to develop force models incorporating such considerations as cutter runout, spindle tilt, and system deflections.

In the integration of local cutting forces, both numerical integration and analytical methods have been documented. While numerical integration [2,3,6] is a general, convenient and effective cutting-force estimation method, analytical integration provides an explicit cutting-force expression to enable process optimisation, machine-tool design and on-line control. Bayoumi et al. [9,10] achieved a closed-form expression for cutting forces for helical milling operations in the time domain by mathematical integration. The change of cutting engagement calls for the use of different integration limits in various cutting segments, therefore different formulae for cutting forces were presented for various cutting angles. Yellowley [11] gave analytical expressions of torque and forces via the integration of the local cutting force of straight cutting edge represented by a Fourier series. The model was developed for either single helical flute or multiple straight flutes, while the general case of multiple helical cutting edges was not addressed. Liang and Wang [12] developed a convolution integration method by which an explicit expression for dynamic cutting-force components could be obtained. The resulting model was given in the frequency domain and it is limited to the end-milling operation.

In this paper, a generalised model is developed to provide analytical, explicit, and algebraic expressions for cutting forces applicable to both end-milling and face-milling processes involving multiple helical flutes. It involves the representation of a generalised cutting-edge function, integration of local cutting forces, formulation of dynamic cutting-force components, and reconstruction of cutting-force waveforms. The method used here avoids the search for integration limits of different cutting engagement segments, and generates a single expression for cutting forces that covers the entire cutter rotation range. The dynamic cutting forces in the model are explicitly expressed in terms of material properties, tool geometry, cutting parameters, and process configuration. The explicit expression provides an advantage over the commonly used method of numerical integration for machine-tool vibration analysis, process optimisation, machine-tool design, and on-line diagnostics. The development of this model is based on a generalised cutting-edge function rather than for any specific
cutter geometry and process configuration. Therefore, the model is generic and can be used to develop a cutting force model for any milling operation with known cutter geometry. The cutting-edge functions and cutting-force models for both face milling and end milling are presented as degenerated cases of the generalised cutting force model. Experiments are performed and results presented to verify the analytical models.

2. Model Development

2.1 Cutting-Edge Representation

A cylindrical coordinate system is used in this study to represent a cutting edge, as shown in Fig. 1. Suppose the H direction is in the direction of the angular velocity of the spindle, the generalised cutting engagement in terms of height $H$ and radius $R$ of cutting points in continuous and multitooth cutting can be denoted as:

$$h = h(\beta) \ast Tth(\beta), \quad r = r(\beta) \ast Tth(\beta)$$

where \( Tth(\beta) = \sum_{i=1}^{\infty} \delta \left( \beta - \frac{2\pi(i-1)}{N} \right) \) (1)

Note that the tooth sequence function, \( Tth(\beta) \), is a train of unit impulses having the period of the tooth spacing. Since a periodic function can be represented as a convolution of a non-periodic function and an impulse train, the periodicity of the cutting process is herein represented as the convolution of a cutting-edge function and the tooth-sequence function.

![Fig. 1. Cutting edge representation in both cylindrical and cutting-force coordinate systems.](image1)

![Fig. 2. Local cutting forces and cutting configuration.](image2)