Biophysik 10, 89—98 (1973)

On the Generation of Potential Differences across the Membranes of Ellipsoidal Cells in an Alternating Electrical Field*

J. Bernhardt and H. Pauly

Institut für Physikalische und Medizinische Strahlenkunde der Universität Erlangen-Nürnberg
(Direktor: Prof. Dr. med. Dr. phil. nat. H. Pauly)

Received February 10, 1973

Summary. Particles with a nonconducting membrane, oriented in an alternating electrical field, will show the behaviour of electrical dipoles. Across the membranes there will be generated alternating electrical potential differences, which may be calculated for confocal ellipsoidal cells by solving Laplace's equation. We have evaluated a formula valid generally for single confocal ellipsoidal cells under physiological conditions, the cells being placed with one of their semi-axes parallel to an external electrical field. The values of the generated potential difference, considered at the position of their maximum values, are dependent on the shape and size of the cells, on their orientation to the electrical field and on the frequency and strength of the field. The relaxation frequency depends also on cell shape, size and orientation, but furthermore on the membrane properties and on the conductivities inside and outside the cells. For simple cases like spheres and cylinders perpendicular to the electrical field, our formula will correspond to known expressions. Values for the generated potential differences, form-factors and relaxation frequencies are given for different types of spheroids and at different orientations. Of some practical importance are long prolate spheroids with their long semi-axes parallel to the external field, because only small field strengths are necessary in order to generate large potential differences which may evoke action potentials *e.g.* in muscle or nerve cells. The significance of this mechanism concerning the determination of protection and safeguard standards for the exposure to low-frequency electrical fields is discussed.

1. Introduction

This theoretical paper deals with nonthermal effects of alternating electrical fields on cells in a frequency range in which the stimulating effects of electrical currents are of importance. Generally, particles with a nonconducting shell (membrane), oriented in a low-frequency alternating electrical field, will show the behaviour of electrical dipoles. Across the membranes alternating potential differences will be generated. If the depolarization of the membrane resting potential is sufficiently large, action potentials in nerve or muscle cells may be evoked. Whether the action potentials really appear depends on a threshold amount of charge which must flow through the cell membrane — *e.g.* during the half-period of the generated voltage.

In this paper we investigate only the generated potential differences which are dependent on the external field strength, on shape, size and orientation of the single cells, and on the conductivity inside and outside the cells. The generated potential differences may be calculated by solving Laplace's equation. Solutions

* Dedicated to Prof. Dr. Dr. h. c. mult. B. Rajewsky on the occasion of his 80th birthday.

are known for spherical and cylindrical particles. But from the standpoint of mathematical generality solutions of Laplace's equation for ellipsoids with shell are more interesting and more flexible in practice too. The electrical properties of ellipsoidal cells suspended in a liquid were investigated thoroughly by Fricke [1 to 5], and also by Velick and Gorin [14]. Fricke [4] stated a formula for the generated potential difference which is valid for very low frequencies. The form-factor in Fricke's formula depends on shape and orientation, but not on the size of the ellipsoids, and was calculated for different axial ratios by Velick and Gorin [14]. This paper deals more with the frequency behaviour of the generated potential difference. Here two further form-factors are occurring determining the relaxation frequency. These are depending not only on shape and orientation but also on the size of the ellipsoids. Prolate spheroids with their long semi-axes parallel to the external field are of some practical importance because the generated potential differences are especially large. The significance of the latter case to the determination of protection and safeguard standards for the exposure to low-frequency electrical fields is discussed.

2. Results

2.1 General Formulas

Solutions of Laplace's equation in elliptical coordinates were calculated and applied to ellipsoids with a shell which are placed in an electrical parallel field. In this case an analytical solution of Laplace's equation is only possible for confocal ellipsoids. This will without doubt limit the range of application of the formulas obtained. The aim of the calculations is the evaluation of the generated potential difference across the membrane at the position of its maximum value, and furthermore its dependence on the external field, on shape, size and orientation of the ellipsoid and on the electrical properties inside and outside.

For pure and isotropic dielectrics the general potential functions inside and outside a full ellipsoid with whatever orientation in a static electrical field were calculated by Stratton [13]. An ellipsoid with shell, one of the three semiprincipal axes a_1, a_2, a_3 parallel to the external field, is presented in Fig. 1 in a coordinate system x_i. The equation

$$\sum_{i=1}^{3} \frac{x_i^2}{a_i^2 + \xi} = 1$$

represents a family of confocal ellipsoids in this coordinate system. If a_k is the smallest semi-axis, then the points of the interior of the ellipsoid are given by $-a_k^2 < \xi < \xi_1$, the points in the shell are given by $\xi_1 \leq \xi \leq 0$ and the points outside the shell by $\xi > 0$. ξ_1 may be calculated from $\xi_1 = -2a_i d_i + d_i^2$ ($i = 1; 2; 3$), where d_i is the thickness of the shell in the x_i direction.

The potential function of the shell was obtained from a linear combination of the potential functions of the full ellipsoid. The unknown constants were calculated as usual from the boundary conditions. In general we cannot assume the system to be purely dielectric, but must take into account an electrical conductivity $\varkappa$. In this case the distribution of the electrical field in the system is determined by the complex dielectric constants $\varepsilon^* = \varepsilon - j \frac{\varkappa}{\varepsilon_r \omega}$ of the body and of the surrounding