On the micromechanics theory of Reissner-Mindlin plates

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Summary. A micromechanics model is developed for the Reissner-Mindlin plate. A generalized eigenstrain formulation, i.e., an eigencurvature/eigen-rotation formulation, is proposed, which is the analogue or counterpart of the eigenstrain formulation in linear elasticity. The micromechanics model of the Reissner-Mindlin plate is useful in the study of mechanical behavior of composite plates that contain randomly distributed inhomogeneities, whose sizes are close to the order of thickness of the plate; under those circumstances, the use of micromechanics of linear elasticity is not justified, and, moreover, it is inconsistent with structural theories, such as the Reissner-Mindlin plate theory, that are actually used in engineering design.

In this paper, the analytical solution of an elliptical inclusion embedded in an infinite thick plate is sought. In particular, the first order asymptotic (or approximated) solution of the elliptical inclusion problem is obtained in explicit form. Accordingly, the Eshelby tensors of the Reissner-Mindlin plate are derived, which relate eigencurvature and eigen-rotation to the induced curvature and shear deformation fields. Several variational inequalities of the Reissner-Mindlin plate are discussed and derived, including the comparison variational principles of Hashin-Shtrikman/Talbot-Willis type. As an application, variational bounds are derived to estimate the effective elastic stiffness of Reissner-Mindlin plates, specifically, the flexural rigidity and transverse shear modulus. The newly derived bounds are congruous with the Reissner-Mindlin plate theory, and they provide an optimal estimation on effective rigidity as well as effective transverse shear modulus for unstructured composite thick plates.

1 Introduction

In this paper, we are concerned with a micromechanics model of Reissner-Mindlin plates, which is an important subject in engineering practices, and, to the author's knowledge, it has been neglected both in mechanics literature and engineering design. Part of the reason for such oblivion is that in the past the term, "composite plate", is only referred to the multiphase plates that have definite structures, such as laminar plates, or well structured lattice plates (e.g., Christensen [4], Mindlin [28], and Kaprielion et al. [20]). Today, many composite plates are made of materials with randomly distributed heterogeneous constituents or unstructured composite materials, and therefore such "oblivion" becomes inexcusable. Recently, a micromechanics model of the Love-Kirchhoff plate is proposed by Li [26]. This study is a further development of micromechanics in the framework of structural mechanics, in an attempt to compound a systematic exposition of structural micromechanics, which is in parallel with the micromechanics in linear elasticity.

The micromechanics of linear elasticity theory rests upon the notion of representative volume element (RVE) (Hill [16], Hashin [15], Kröner [22], [23], Willis [49], and Nemat-Nasser and Hori [32]), which is essentially a provision on the length scale of the aggregates, within
which the micromechanics theory is valid in a statistical sense. For a composite plate, in which that characteristic size of the micro-element is only one order magnitude less than or at the same order of the thickness of the plate, the application of conventional micro-elasticity will become questionable. Furthermore, since the conventional micro-elasticity theory does not carry any information about structural mechanics, if a structure’s effective stiffness, such as flexural rigidity or transverse shear modulus, is evaluated by linear micro-elasticity, it is certainly not compatible with the theory of structure mechanics that is used in actual strength analysis. Strictly speaking, the constitutive equations of Reissner-Mindlin plates, i.e., the relations between moment/resultant and curvature/rotation, are derived by taking into account additional internal constraints on the constitutive laws of linear elastic materials (e.g., Naghdi [30], [31]). Thus, the constitutive relations at the structural level are fundamentally different from the constitutive relations at continuum level. For instance, a Reissner-Mindlin plate is intrinsically anisotropic in a mathematical sense, even if the matrix material of the plate is isotropic. It is, therefore, erroneous in principle to evaluate the effective stiffness of the Reissner-Mindlin plate based on the micromechanics of linear elasticity theory.

From this standpoint, developing micromechanics models in structure mechanics can be instrumental in the engineering design analysis. To pursue such a novel scheme, the first logical step seems to be replacing the notion of the representative volume element by the notion of the representative area element (RAE): An representative area element defined for a material point in a two-dimensional (2-D) manifold is a material element which is a statistically representative of all the material points in a material neighborhood at a specified scale on the two-dimensional manifold. The continuum material point is called meso-area-element, whereas its micro-constituents are called micro-area-elements. An RAE must include a very large number of micro-area-elements, in order that the representative information is statistically stable. However, in three-dimensional (3-D) space, there is no physical object that is truly a 2-D mathematical manifold. An representative area element is actually a special representative volume element whose properties are homogeneous, in an average sense, in the direction that is perpendicular to the surface area of every micro-area-element. For a Reissner-Mindlin plate, it implies that the material concentration of every micro-area-element dominates statistically in the thickness direction. This is consistent with the Reissner-Mindlin plate theory, because the theory of Reissner-Mindlin plates is constructed through a special “homogenization” process (or averaging process) of 3-D elasticity theory in the thickness direction of the plate. Figure 1 illustrates an ideal model of such a representative area element. In reality, the inhomogeneous phases do not need to penetrate through the thickness of the plate, since it is only required that the concentrations of every species dominate statistically in the thickness direction.

In Reissner-Mindlin plate theory, two rotational degrees of freedom are assigned at each material point on the plate’s middle surface, and the rotation vector lies on the plane of the