Direct Superposition of Wilson Trial Functions
by Computerized Symbolic Algebra

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Summary

Method of direct superposition of trial vectors, proposed by Wilson, is elucidated
for the vibration analysis of systems, possessing damping, by the computerized symbolic
algebra. The essence of the method is using a specific set of trial functions (Wilson trial
functions) derived in a special manner from the appropriate static solution, rather than
performing a mode superposition analysis by the exact eigenvectors of the system. Im-
mediate advantage of the method is that the static solution, to which a dynamic solution
should tend for the vanishing excitation frequency, is obtained automatically, by using
a single vector, whereas within the exact eigenvectors, infinite number of eigenvectors
are involved to obtain a static solution. A specific example is numerically evaluated and
it is clearly demonstrated that the superposition of the Wilson trial functions yields ex-
tremely accurate results with fewer vectors than using the conventional set of trial func-
tions, utilized within the Rayleigh-Ritz method.

1. Introduction

Usually the analysis of the forced vibration of the damped system is preceded
by the free vibration study, namely by the evaluation of the natural frequencies
and the mode shapes. Then mode superposition analysis is performed, where the
given (excitation) and sought (response) functions are expanded in terms of the
mode shapes of the undamped structure. As is well recognized, the numerical
determination of the exact natural frequencies and mode shapes can require a
large numerical effort. The usefulness of the prior knowledge of the natural
frequencies lies in that one can forecast the possible resonant conditions, since in
the vicinity of natural frequencies the magnification ratios assume considerable
values. Modal superposition technique may require however large amount of
modes to be taken into account to accurately predict the structural response.
For example, it is well recognized that to capture the static load effects, especially
for concentrated loads, a considerable amount of eigenvectors can be required \cite{1}, \cite{2}. Wilson et al. \cite{3}, \cite{4} proposed a new method which overcomes the above predicament which may arise with using the exact eigenvectors. The use of the alternative set of orthogonal vectors, which are not the eigenvectors of the system, provides accurate solution at a reduced computational cost. The main idea of Wilson’s method is as follows.

The equations of motion of the system (written on terms of finite elements) read

\[ M\ddot{u} + C\dot{u} + Ku = f(s) \quad r(t), \quad (1) \]

where \( M, C \) and \( K \) are the mass, damping and stiffness matrices respectively. The vector \( f(s) \) represents the spatial distribution of the loading for fixed \( t \), whereas \( r(t) \) is a temporary distribution for fixed \( s \). The first Ritz vector is found from the solution of the static problem

\[ Ku_1^* = f(s). \quad (2) \]

We perform then the normalization with respect to the mass matrix:

\[ u_1 = \frac{u_1^*}{\sqrt{u_1^{**T}Mu_1^{**}}}, \quad (3) \]

so that

\[ u_1^{T}Mu_1 = 1. \quad (4) \]

The subsequent vectors are generated from the following recurrence relationship

\[ Ku_i^* = Mu_{i-1}, \quad i = 2, 3, \ldots, N. \quad (5) \]

The vectors are orthogonalized at each step by the use of the procedure

\[ u_i^{**} = u_i^* - \sum_{j=1}^{i-1} c_j u_j \quad (6) \]

where

\[ c_j = u_j^{T}Mu_j^*. \quad (7) \]

The vectors are then normalized, in perfect analogy with Eq. (3):

\[ u_i = \frac{u_i^{**}}{\sqrt{u_i^{**T}Mu_i^{**}}}. \quad (8) \]

As a result the set of functions \( u_i \) is orthonormal

\[ u_i^{T}Mu_j = \delta_{ij}, \quad (9) \]

where \( \delta_{ij} \) is Kronecker’s delta.

The vectors so generated are used for the solution of the forced vibration problem. Such a procedure automatically captures the static response problem, since the first vector is derived from the static solution.