Global optimization using accurate approximations in design synthesis

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Abstract The design variable space of a design synthesis problem may contain multiple local optima. In the approximation concepts approach to design synthesis, the design objective and constraint functions are approximated in order to reduce the overall cost. If the approximations accurately capture the actual behavior of the objective function and constraints, then the approximate design variable space may also contain local optima. In this work, a multistart optimization algorithm is used to search for the global optimum of the actual design using just a few design cycles. Example problems are presented to illustrate the methodology set forth.

1 Introduction

In the approximation concepts approach to design synthesis (Schmit and Farshi 1974; Schmit and Miura 1976), the actual design is first analyzed to calculate the design objective function and constraint values. Then, a sensitivity analysis is performed to calculate the gradients of the objective function and constraints with respect to the design variables. The objective function and constraint values and their gradients are used to form an approximate analysis problem. This approximate analysis problem is then used in conjunction with a nonlinear optimization algorithm to find an approximate optimum design. This optimum design is used for the next analysis and sensitivity analysis of the actual design. The procedure is then repeated until convergence (see Fig. 1). The sequence of approximate optimum designs leads to a local optimum in the actual design variable space in most cases.

If the actual space is convex and linear, and convex approximations of the objective function and constraints are formed using a first order Taylor Series with respect to the design variables, the sequence of approximate optima will lead to the actual global optimum.

If the Taylor Series is expanded with respect to the design variables or their reciprocals, based on the sign of the gradient derivatives, the approximation is said to be hybrid (Starnes and Hafkka 1979). It can be shown that this type of approximation forms a convex approximate design space (Fleury and Schmit 1980). If approximations of the objective function and constraints are formed using hybrid approximations, then the sequence of approximate optima will lead to a local optimum in the actual design variable space.

More accurate approximations may be formed using the concepts of intermediate design variables (Yoshida and Vanderplaats 1988; Vanderplaats and Salajegheh 1988) and intermediate response quantities (Bofang 1987; Vanderplaats and Salajegheh 1989; Hansen and Vanderplaats 1990; Vanderplaats and Han 1990; Canfield 1990; Thomas et al. 1990;
2 Multistart optimization algorithm

There are two types of algorithms to solve for the global optimum of nonconvex optimization problem, namely deterministic and stochastic. Deterministic algorithms usually guarantee convergence to the global optimum only for certain classes of objective and/or constraint functions. Stochastic algorithms randomly generate points in the design space to initiate local searches, and they only guarantee convergence to the global minimum asymptotically, that is, roughly speaking, the probability that the algorithm finds the global optimum converges to one as the number of local searches tends to infinity. Multistart methods are stochastic and enumerate all local minima with local optimizations initiated from a set of random initial designs distributed uniformly over the feasible domain. The basic steps are: (Rinnooy-Kan and Timmer 1986)

1. Select a design from a uniform distribution on the feasible domain.
2. Starting from this design, use a local optimizer to find the associated local minimum.
3. Check stopping rules. If a termination criterion is satisfied, stop sampling (generating initial designs for local optimization) and the local minimum with the lowest objective function value estimates the global minimum. If the termination criterion is not satisfied, go to step 1.

The main difficulty in implementing a multistart algorithm is to derive efficient stopping rules, that is, deciding when to stop the sequence of local searches. Since stochastic in nature, multistart algorithms do not necessarily find the global minimum, but the probabilistic structure provided by the sampling distribution for the starting points allows to make inferences about problem parameters such as the number of local minima and the relative sizes of the regions of attraction for each one of them. With these estimates, the process is stopped when the number of local minima found is equal to the estimated number of local minima or when the relative size of the non-visited regions of attraction is smaller than a prescribed value (Boender and Ronnooy-Kan 1987).

A second type of stopping rules is derived using elements of statistical decision theory. For this purpose, a cost is associated with each local search and the process is then stopped when the expected benefit of initiating a new local search is smaller than its cost. One of the most efficient rules in this context is derived in Betro and Schoen (1987). This rule allows the user to express the cost of each local search in the same units of the objective function, usually a given percentage of the lowest objective found so far. In simpler terms, the cost represents the minimum expected reduction of the objective if a new local search is to be initiated.

The stopping rules used in the computer implementation used in this paper are:

1. The expected relative size of the detected regions of attraction is given by
   \[
   \frac{(n - w - 1)(n + w)}{n(n - 1)},
   \]
   where \( n \) is the number of local searches and \( w \) is the number of distinct local minima found so far. The algorithm stops performing new local searches as soon as the relative size of the detected regions of attraction exceeds 99%.

2. A loss function expressing the cost associated with stopping after \( n \) local searches is defined as
   \[
   L(t_1, \ldots, t_n; c) = t(n) + nc,
   \]
   where \( t(n) \) is the lowest objective function value found so far and \( c \) is the cost of a local search expressed in units of objective function. In this paper, \( c \) is taken as 0.1% of \( t(n) \).

The stopping rules prescribe to stop sampling when the present cost is lower than the total expected cost of a new local search. Using Bayesian analysis, these rules translate to Betro and Schoen (1987):

for \( t(n) > a \)

\[
\lambda \frac{\exp(-S)}{n + \lambda} \left\{ -a - 0.0429b + t(n) + 0.58565b \exp\left( (a - t(n))/b \right) \right\} \leq c,
\]

for \( t(n) \leq a \)

\[
\frac{\lambda}{n + \lambda} \exp(-S)0.0036b \exp\left[ -0.8284(a - t(n))/b \right] \leq c,
\]

where

\[
S = \sum_{j=1}^{n} \left\{ \gamma(t(j-1)) - \gamma(t(j))/(m(j-1) + \lambda) \right\},
\]

\[
1 - F_0(t) = \begin{cases} 
1 - 0.7071e^{(-a+1)/b} & t \geq a \\
0.5e^{-0.8284(a-1)/b} & t < a
\end{cases}
\]

\( \lambda \) is a positive parameter which, as suggested in Betro and Schoen (1987), is taken as \( \lambda = 1 \) in this paper.

The parameters \( a \) and \( b \) are assessed by the user, considering that the probability of an objective value lower than \( a - 4.07b \) and the probability of an objective value higher than \( a + 4.72b \) are both 0.01.

The reason for using these stopping rules is to bound the number of local searches in the multistart process. In fact, for stopping rule 1 the number of local searches is linear in the