DISTRIBUTION OF RADIANT THERMAL FLUX
OVER THE SURFACE OF A SPHERE FOR HYPERSONIC
FLOW OF A NONVISCOUS RADIATING GAS PAST
THE SPHERE

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In the present study using the Newtonian approximation [1] we obtain an analytical solution to
the problem of flow of a steady, uniform, hypersonic, nonviscous, radiating gas past a sphere.
The three-dimensional radiative-loss approximation is used. A distribution is found for the
gasdynamic parameters in the shock layer, the withdrawal of the shock wave and the radiant
thermal flux to the surface of the sphere. The Newtonian approximation was used earlier in
[2, 3] to analyze a gas flow with radiation near the critical line. In [2] the radiation field was
considered in the differential approximation, with the optical absorption coefficient being
assumed constant. In [3] the integrodifferential energy equation with account of radiation was
solved numerically for a gray gas. In [4-7] the problem of the flow of a nonviscous, nonheat-
conducting gas behind a shock wave with account of radiation was solved numerically. To
calculate the radiation field in [4, 7] the three-dimensional radiative-loss approximation was
used; in [5, 6] the self-absorption of the gas was taken into account. A comparison of the
equations obtained in the present study for radiant flow from radiating air to a sphere with the
numerical calculations [4-7] shows them to have satisfactory accuracy.

1. We consider the hypersonic flow of a nonviscous, nonheat-conducting radiating gas past a spheri-
cal body of radius R. The system of equations that describes the gas flow between the withdrawing shock
wave and the sphere, written in a spherical coordinate system fixed in the body, is given in [5]. This sys-
tem of equations is solved with boundary conditions on the oblique discontinuity and with the condition that
there is no flow on the body. It is further assumed that the gas is ideal, the shock layer is thin, and the
conditions of the hypersonic approximation are satisfied, so that

\[ p_\infty \ll \rho_\infty V_\infty^2, \quad h_\infty \ll V_\infty^2/2 \]

where \( p_\infty \) is pressure, \( \rho_\infty \) is density, \( h_\infty \) is enthalpy, and \( V_\infty \) is velocity of the incident flow.

The gas satisfies the equation of state

\[ h = \frac{\gamma}{\gamma - 1} \frac{p}{\rho} \]  \hspace{1cm} (1.1)

where \( \gamma \) is the effective ratio of specific heats behind a compaction discontinuity, \( p \) is pressure, \( h \) is
enthalpy, and \( \rho \) is density.

To solve the problem we expand quantities in the small parameter \( \varepsilon \), equal to the ratio of the gas
density before and after the shock wave (Newtonian approximation) [1]

\[
(r - R)/R = \varepsilon y_0 + \ldots, \quad u = V_\infty [u_0 + O(\varepsilon)], \quad v = V_\infty [v_0 + O(\varepsilon^2)]
\]

\[
p = \rho_\infty V_\infty^2 [p_0 + O(\varepsilon)], \quad \rho = \rho_\infty \omega_0 [p_0 + O(\varepsilon)], \quad h = \frac{V_\infty^2}{2} [h_0 + O(\varepsilon)]
\]

\[
Q = \frac{\sigma}{\varepsilon R} \left( \frac{V_\infty^2}{2k_R} \right)^\mu [Q_0 + O(\varepsilon)]
\]

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Here $r$ is radial coordinate, $u$ is tangential velocity component, $v$ is normal velocity component, $p$ is pressure, $h$ is enthalpy, $\rho$ is density, $\kappa$ is the Stefan–Boltzmann constant, $R$ is the radius of the body, $Q = \text{div} \, \text{qR}$ is the divergence of the radiant flow, $\varepsilon = (\gamma - 1)/(\gamma + 1)$, and $C_p$ is the effective specific heat at constant pressure.

The subscript $\infty$ indicates quantities in the incident flow, the subscript $S$ indicates quantities on the shock wave, and the subscript 0 indicates dimensionless quantities.

Equations (1.2) are substituted into the equations of gasdynamics, after which we obtain for the first terms of the expansion (the subscript 0 is dropped)

$$\frac{\partial}{\partial y} \rho v + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \rho u) = 0, \quad \rho u^2 = \frac{\partial p}{\partial y}$$

$$\rho v \frac{\partial u}{\partial y} + \rho u \frac{\partial v}{\partial \theta} = 0, \quad \rho v \frac{\partial h}{\partial y} + \rho u \frac{\partial h}{\partial \theta} = - \Gamma Q$$

$$\frac{h}{\gamma - 1} \frac{p}{\rho}, \quad \Gamma = \frac{2\kappa V_\infty}{\rho_\infty} \left( \frac{V_\infty}{2C_p} \right)^4$$

Here $\Gamma$ is the radiation parameter [5] and $\theta$ is the angular coordinate.

To the same approximation the relations on the shock wave take the form

$$u_s = \sin \theta, \quad v_s = - \cos \theta, \quad p_s = \cos^2 \theta, \quad h_s = \cos^2 \theta$$

The condition of nonflow of the body takes the form

$$v(y = 0) = 0$$

We determine the dimensionless stream function

$$d\psi = \rho u \sin \theta \, dy = \rho v \sin \theta \, d\theta$$

Further, in system (1.3) we transform to the variables $\Psi = \theta$. Then in these variables the system (1.3) is written as follows:

$$\frac{\partial u}{\partial \theta} = 0$$

$$\frac{\partial p}{\partial \Psi} = u \sin \theta$$

$$\frac{\partial y}{\partial \Psi} = 1/\rho u \sin \theta$$

$$v = u \frac{\partial y}{\partial \theta}$$

$$\rho u \frac{\partial h}{\partial \theta} = - \Gamma Q$$

On the body we assume the dimensionless stream function $\Psi = 0$, on the shock wave $\Psi = \Psi_S(\theta) = 1/2 \sin^2 \theta$.

The boundary conditions (1.4) in new variables have the same form, but they must be taken for $\Psi = \Psi_S(\theta)$.

2. The solution of Eq. (1.7) with boundary conditions (1.4) has the form

$$u = \sqrt{2\Psi}$$

Taking account of this solution, the expression for the pressure from Eq. (1.8) with account of (1.4) takes the form

$$p(\Psi, \theta) = \cos^2 \theta - \frac{1}{3} \sin^2 \theta + \frac{(2\Psi)^{1/2}}{\sin \theta}$$

For the geometrical coordinate we obtain the expression

$$y(\Psi, \theta) = \frac{1}{\sin \theta} \int_0^\Psi \frac{d\Psi}{\rho \sqrt{2\Psi}}$$

Equation (1.10) determines the normal velocity component, which satisfies the boundary condition (1.4). For the final solution of the gasdynamic problem we must solve Eq. (1.11) with boundary condition (1.4). In the three-dimensional radiative-loss approximation the divergence of the radiant flow has the form (in dimensional form) [7–9]

$$Q = 2 \mu \sigma T^4$$