AN IMPROVED BOUNDARY ELEMENT GALERKIN METHOD FOR THREE-DIMENSIONAL CRACK PROBLEMS

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Dedicated to Prof. Dr.-Ing. W. L. Wendland on the occasion of his 50th birthday.

In this paper we analyze the solution of crack problems in three-dimensional linear elasticity by equivalent integral equations of the first kind on the crack surface. Besides existence and uniqueness we give sharp regularity results for the solution of these pseudodifferential equations. Two versions of Eskin's Wiener-Hopf technique are presented: the first one requires the factorization of matrix-valued symbols which is avoided in the second case. Based on these regularity results we show how to improve the boundary element Galerkin method for our integral equations by using special singular trial functions. We apply the approximation property and inverse assumption of these elements together with duality arguments and derive quasi-optimal asymptotic error estimates in a scale of Sobolev spaces.

1. INTRODUCTION

Crack problems play an important role in engineering applications. They present difficulties for both the mathematical analysis and the numerical approximation due to the fact that their solutions have singularities at the crack front. The present paper is concerned with the precise description of these singularities and with approximation schemes (based on boundary integral equations) using singular trial functions.

We use boundary integral equations of the first kind for solving the Dirichlet and Neumann crack problems in three-dimensional linear elasticity. Existence, uniqueness, and regularity of the solutions of the integral equations are analyzed

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using the calculus of pseudodifferential operators and the Wiener-Hopf technique developed by Eskin [5]. So far, Eskin's results and their improvements in [9] have been only applied to problems which can be reduced to scalar problems.

Here we encounter some of the peculiar features of matrix-valued problems. We give two methods for their analysis: One is based on the factorization of $2 \times 2$-polynomial matrices and the other on a special reduction to scalar equations, following an idea from [10]. With both methods we obtain a decomposition of the exact solutions into regular and singular parts. We give a-priori estimates in Sobolev norms for the regular parts and for the stress intensity coefficients in the singular parts. The estimates in Theorems 3.1 and 3.2 improve the regularity results in [10].

For the approximation schemes the trial functions in our boundary element Galerkin methods are composed of two-dimensional regular finite element functions and of special singular elements which are defined by approximations of the stress intensity coefficients by one-dimensional finite elements. This method was proposed in [10].

We show convergence of the method and give asymptotic error estimates in various Sobolev norms (see Theorem 4.6). As usual in boundary element methods [15] the error analysis makes use of the following tools: strong ellipticity of the boundary integral operators, a priori estimates for the exact solutions, approximation properties and inverse assumptions for the augmented boundary element spaces and, finally, Aubin-Nitsche type duality arguments.

The paper is organized as follows:

In Section 2 we derive the first kind boundary integral equations on the crack surface. The kernels are weakly singular for the Dirichlet problem and hypersingular for the Neumann problem, and they define strongly elliptic pseudodifferential operators of order $-1$ and $1$, respectively. We show unique solvability of the integral equations in the respective energy spaces (see Theorem 2.4).