Abstract. The effects of system parameter uncertainties on system performance are always of great concern to the system designer. It is desirable to know a priori estimates of the system response, subject to parameter uncertainties. In the following we propose using interval analysis techniques to establish estimates of system performance (e.g., envelopes of time response and frequency response). The results which we obtain constitute generalizations of existing work. Specifically, existing results address systems which are described by linear ordinary differential equations which are endowed with a single parameter belonging to an interval. In the present results we address systems described by linear or nonlinear ordinary difference equations endowed with more than one parameter belonging to intervals.

1. Introduction

In an attempt to account for finite word-length effects in digital computers, Moore [1], [2] and others introduced interval arithmetic and interval analysis methods. Subsequently, these methods have been used in a variety of applications, including solutions of nonlinear circuit equations [6], two-point boundary-value problems [8], linear programming [9], computation of eigenvalues [10], worst-case analysis of linear circuits [7], determination of envelopes of the response of a linear system endowed with a single parameter belonging to an interval [3]–[5], [13], and other applications.

The changes in the performance of a dynamical system due to the changes in system parameters are of great practical importance in engineering analysis and design. Such perturbations of parameters may model the effects of uncertainties in manufacturing tolerances, “aging” of components, environmental causes and the like. Because of these uncertainties, the values of the

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parameters of a given dynamical system may frequently be viewed as belonging to suitable intervals. In doing so, the system description is then phrased in terms of differential equations, or difference equations involving parameters that belong to appropriate intervals.

In [3]-[5] and [13] a complete metric space of continuous interval functions of an interval variable was constructed which includes the \(C([a, b])\) real-valued function space and the united extension (defined later) of its member functions \(f\), denoted by \(\hat{f}\). It is proved that the rational interval functions with an interval variable (say, \([a, b]\)) also belong to this space and exhibit the inclusion property \(f([a, b]) \supseteq \hat{f}([a, b])\). In [3]-[5] and [13] a theorem is established which asserts that by using a sufficiently fine partition of the interval \([a, b]\), it is possible to approximate the exact range of an interval function \(f([a, b]), \hat{f}([a, b])\), as closely as desired, by computing the union of the range of each of the interval functions determined by this partition.

The results in [3]-[5] and [13] address continuous-time, finite-dimensional, linear dynamical systems endowed with one uncertain parameter belonging to a specified interval. What we present in this paper are some new general fundamental interval analysis results which enable us to estimate solution bounds for initial-value problems which (a) are endowed with more than one parameter belonging to more than one interval, (b) need not be linear, and (c) are applicable to continuous-time and to discrete-time systems.

The principal theorem is presented in Section 4. Section 2 presents the notation for the subsequent sections, while Section 3 contains the definitions for interval arithmetic and preliminaries which constitute the necessary background for the principal theorem of Section 4. In Section 5 we present a specific example to demonstrate the applicability of the present results. The paper is concluded in Section 6.

2. Notation

Let \(V\) and \(W\) be arbitrary sets. Then \(V \cup W, V \cap W, V - W\), and \(V \times W\) denote the union, intersection, difference, and Cartesian product of \(V\) and \(W\), respectively. If \(V\) is a subset of \(W\), we write \(V \subset W\). We say \(V\) and \(W\) are equal, written as \(V = W\), if \(V \subset W\) and \(W \subset V\). For any \(x\) belonging to \(V\), we write \(x \in V\) and we call \(x\) a point or an element of \(V\). If \(f\) is a mapping from \(V\) into \(W\), we write \(f: V \to W\) and \(f(U) = \{f(x) \in W | x \in U\}\) for \(U \subset V\), and \(f^{-1}(y) = \{x \in V | f(x) = y\}\) for \(y \in W\). Let \(R\) denote the set of real numbers and let \(R^+\) denote the set of nonnegative real numbers.

An interval, \(I\), is defined to be a subset of the real numbers of the form

\[ I = \{x \in R | a \leq x \leq b \text{ with } a, b \in R\}. \]

We denote \(I\) as \([a, b]\). If \(b = a\), the interval \([a, a]\) is called a degenerate interval. Two intervals \(I\) and \(J\) are said to be equal if they are equal in the set