A closed system of equations is derived for the energy flux, and the boundary conditions are given. The transport coefficients and other parameters are found from elementary gaskinetic considerations for a high concentration of the solid phase. As an example, the solution is found for the problem of an "adiabatic" Couette flow for a granulated medium.

1. The hydrodynamics of multiphase systems does not yet have a satisfactory theoretical basis despite the fact that it is of wide practical application and that a very considerable amount of experimental data has been accumulated [1]. The averaged equations of motion of two-phase streams are not closed and can be applied to nonuniform flows provided that the dimensions of the elements in the dispersed phase are much smaller than the characteristic size of the channel or boundary layer.

The equations of averaged motion of the phases can be obtained in closed form if the interaction mechanism is known. Two limiting situations can be distinguished: the concentration of the dispersed phase is small, and its elements interact only with the carrier medium (gas, liquid); the concentration of the dispersed phase is close to maximum, and the movements of the elements have an order which is significantly smaller than their characteristic size.

It is the latter type of system that we consider in this paper (a generalization of [2]). We take a two-fluid model described by the phenomenological hydrodynamic equations in which the transport coefficients and other quantities are derived from elementary kinetic considerations. These considerations are approximate but they do allow the principal relations to be understood and do reflect the main properties of the various phenomena.

We follow a method in which the two phases are described separately and their interaction is taken into account by means of a body force $F_i$. Since the size of the solid particles is taken to be much smaller than the characteristic dimensions of the flow and since their concentration is considerable, the gradient of the "averaged" velocity of the fluid is negligibly small in comparison with the local gradients near the particles and so the equations of motion of the fluid can be written in the Euler form

$$\rho \frac{d\mathbf{v}_i}{dt} = -F_i - \frac{\partial p}{\partial x_i} + \rho \mathbf{g}_i$$

$$\frac{\partial (\rho \mathbf{v}_i)}{\partial t} + \frac{\partial (\rho \mathbf{v}_i \mathbf{v}_i)}{\partial x_j} = 0$$

The force $F_i$ which acts on the stream from the particles is greater than the Stokes term in order of magnitude. Here $\rho$ is the density of the fluid, $\varepsilon$ is the porosity (fraction of the fluid in unit volume), $p$ is the pressure of the fluid phase, $\mathbf{g}_i$ is the acceleration due to mechanical forces, $t$ is the time, and $\mathbf{v}_i$ is the average velocity of the fluid, which is related to the velocity in an empty cross section $v_{0i}$ by the equation $v_{0i} = \varepsilon \mathbf{v}_i$.

For the solid phase, the equations of motion are

$$\rho_s \frac{d\mathbf{w}_i}{dt} = F_i + \frac{\partial \mathbf{v}_{ij}}{\partial x_j} + \rho_s \mathbf{g}_i$$

\[
\frac{\partial (\rho \mathbf{u})}{\partial t} + \frac{\partial (\rho \mathbf{u} \mathbf{u})}{\partial x_i} = 0
\]

where \( \rho \) is the density of the solid phase material, \( \mathbf{u} \) is the volume concentration of the solid phase, \( w_i \) is the average flow speed of this phase, and \( T_{ij} \) is the stress tensor in the solid phase considered as a continuous medium and reflects the interaction of the solid phase particles.

We consider the solid phase to consist of identical spheres of diameter \( d \). The number of particles in unit volume \( n = \frac{6\pi}{d^3} \). Of the forces acting on a particle from the fluid phase, we consider the hydraulic resistance and the transverse Magnus force which acts for a flow round a rotating sphere. We can thus write

\[
f_i = \frac{\pi d^2}{4} \frac{3}{2} \rho \mathbf{u}_i - \frac{\pi d}{6} \frac{1}{\mathbf{u}_i} + \frac{\pi d^2}{3} \rho \mathbf{u}_i \mathbf{w}_i
\]

(1.5)

Here \( \xi \) is the hydraulic resistance coefficient of a particle, \( \omega \) is the angular velocity vector, \( u_i = \omega r_i (v_i - w_i) \) is the maximum flow velocity past the particle as defined by the minimum relative transfer cross section \( \psi \). The expression for the Magnus force has been obtained in [3], and \( \psi = 1 - \frac{1}{1.17} \frac{r^2}{3} \), \( \mathbf{u}_i = \psi^{-1} (v_i - w_i) \) is the average flow velocity round a particle. The approximate relationship \( \psi = 1.09 \cdot (1 - \frac{r^2}{3}) \) can be obtained by a method similar to that used in [2] for calculating \( \psi \).

We assume in the calculation of \( F_i \) that the vector \( \mathbf{w} \) for the particles in unit volume is randomly directed and that therefore the Magnus force, being an internal one, does not enter the momentum equations. Thus

\[
F_i = n \langle f_i \rangle = \frac{1}{d^3} \frac{3}{4} \xi \rho \mathbf{u}_i - \frac{1}{d^3} \xi \rho \mathbf{u}_i \mathbf{w}_i
\]

(1.6)

Equation (1.1) now becomes

\[
\frac{d\mathbf{u}_i}{dt} = -\frac{\partial \rho}{\partial x_i} - \frac{3}{4} \xi \rho \mathbf{u}_i
\]

(1.7)

By comparing (1.7) and (1.1), we can decide about the necessity of including the factor \( \xi \) in front of \( \partial \rho / \partial x_i \).

In order to determine the tensor \( T_{ij} \), we assume that the system of particles can be considered as a fluid which satisfies the Stokes postulates [4]. We can then write for \( T_{ij} \) the general expression [4]

\[
T_{ij} = -\frac{\partial \mathbf{u}_i}{\partial x_j} - \frac{3}{4} \xi \rho \mathbf{u}_i \mathbf{w}_i
\]

(1.8)

where \( D_{ij} \) is the strain rate tensor, \( \rho \), \( \mu \), \( \gamma \) are scalars which depend on the thermodynamic parameters and the invariants of \( D_{ij} \). We introduce the quantity \( p_s = -\frac{1}{3} \rho \left( T_{ii} + \gamma D_{ik} D_{ki} \right) \), which represent the analog of the hydrostatic pressure for a system of particles. Expression (1.8) becomes

\[
T_{ij} = -\left( p_s + \frac{1}{2} \mu \frac{\partial \mathbf{u}_i}{\partial x_j} \right) \delta_{ij} + \mu D_{ii} - \gamma D_{ik} D_{ki}
\]

(1.9)

We assume that as with a normal gas the system has no "memory," i.e., we neglect such effects as second viscosity. This is justified by the fact that the particles do not have internal degrees of freedom except rotational ones and these are considered to be rapidly relaxing.

In order to determine the quantities \( p_s, \mu, \) and \( \gamma \), we take a more definite model of the medium and consider the solid phase as a gas of solid spheres. The distance between particles \( \tau \) was found in [2] as \( l = d(\tau / \tau)^{1/3} - 1 \) on the assumption of similarity between nondense packing with density \( \tau \) and some "standard" dense packing of density \( \tau_0 \).

The particles in the dispersion medium have some "average" motion and also possess random velocities. For a sufficiently dense medium with \( l < d \), a particle can only collide with nearest neighbors and the remaining particles are inaccessible (the phenomenon of "screening"). The mean free path of a particle \( \sim 2l \) and the time between collisions \( t \approx 2l / c v^3 \), where \( c \) is the random velocity of the particles appropriate to one degree of freedom.

The collision time of two spheres is determined by the speed of sound \( c^* \) in the material of the spheres and is equal in order of magnitude to \( t^* \approx 2l / c^* \); thus the fraction of spheres in a collision state is