A method has been proposed [1] for calculating shock-wave adiabatics for two-phase mixtures which does not use the additivity principle, and this is here extended to a medium consisting of \( n \) phases; the shock-wave adiabatic is derived and compared with experiment for a three-phase mixture: an alloy consisting of copper, zinc, and lead (brass). It is shown for a paraffin–tungsten mixture that the adiabatics for the individual phases deviate from ones for the corresponding solid materials. The known equations of state for aluminum and epoxide resin have been used to determine the shock-wave adiabatic for such a mixture. A comparison is made with the analogous calculation made from the additivity principle.

1. Model. Shock waves in multiphase media are involved in research on the propagation of shock waves in rocks impregnated with water or oil [2, 3], in determining the equations of state for one of the phases [1, 4], and in studying the properties of new composite materials and alloys [5, 6]. In most recently published studies [4–7], the shock-wave adiabatic of the mixture has been derived via the additive approximation, in which it is assumed that each phase is compressed in accordance with its own Hugoniot adiabatic for the solid material. However, there is another viewpoint on shock waves in heterogeneous mixtures, which has been presented [1, 8] as that the shock-wave adiabatics of the phases may deviate from those for the pure materials, with the result that at certain phase concentrations one can get an anomalous slope in the Hugoniot adiabatics, as for porous materials [9].

Let the multiphase medium consist of particles differing in nature, the state of each of which is characterized by the pressure, the density, and the temperature; these parameters are related by the equation of state for the corresponding single phase. We assume that the multiphase medium is homogeneous and isotropic on a macroscopic scale. We consider the steady-state propagation of a planar shock wave in such a medium. We assume that the wave amplitude is not too great, so that in the equation of state we do not need to take into account the electronic components for the pressure and internal energy. On the other hand, we assume also that the wave is not so weak that the hydrostatic component of the force field applied to the mixture considerably exceeds the internal shear stresses. Then the medium may be considered as a compressible liquid, and its behavior in response to shock waves is described by the equations of hydrodynamics.

The equations of motion and energy for the mixture [3] are put as follows for an \( n \)-phase mixture if one assumes that the phase pressures are equal (a two-phase mixture was considered in [1]); for phase \( i \) we have the equation of motion

\[
\rho a_i \frac{d \omega_i}{dt} = -a_i \frac{\partial p}{\partial x} + R_i
\]

and the equation of continuity

\[
\frac{\partial (\rho a_i)}{\partial t} + \frac{\partial (\rho a_i \omega_i)}{\partial x} = 0
\]

together with the energy balance equation.
\[ \frac{\partial}{\partial t} \left( \rho_i \varepsilon_i + \frac{w_i^2}{2} \right) + \frac{\partial}{\partial x_i} \left( \rho_i \varepsilon_i + \frac{w_i^2}{2} \right) w_i + a_i p w_i + \frac{\delta W_i}{dt} + q_i = 0 \]

\[ \frac{\delta W_i}{dt} = \rho_i \frac{\partial}{\partial t} + w_i \frac{\partial}{\partial x_i} - \frac{\partial}{\partial x_i} + w_i \frac{\partial}{\partial x_i} \]

Here \( \rho_i \) is the density of phase \( i \), whose concentration by volume is \( \alpha_i \), with \( \sum_{i=1}^{n} \alpha_i = 1 \); \( w_i \) is velocity, \( p \) is pressure, and \( R_i \) is the bulk force from the interaction between phases, which acts on phase \( i \) from the other phases, with \( \varepsilon_i \) the specific internal energy of phase \( i \). \( \delta W_i/\delta t \) the effective work of the forces acting from all other phases on phase \( i \) in unit time, \( q_i \) the flux of heat into phase \( i \) from all the other phases, and \( y_i \) the work of the viscous bulk forces, which is converted to heat, as performed on phase \( i \) by the other phases and arising from relative displacement of the phases. There is a zero contribution overall to the momentum and energy of the whole mixture from the internal interaction between the phases:

\[ \left( \sum_{i=1}^{n} R_i = 0, \quad \sum_{i=1}^{n} q_i = 0, \quad \sum_{i=1}^{n} \delta W_i / dt = 0 \right) \]

The shock-wave speed \( U \) is constant, so it is convenient in (1.1)-(1.3) to convert to the new variable \( z = x - Ut \); we get the continuity integrals for the mass for each of the phases as

\[ \rho_i \partial u_i / \partial t = \partial \rho_i \partial u_i / \partial x_i = M_i \quad (i = 1, 2, \ldots, n) \]

(1.4)

and the integrals for the conservation of the total momentum of the medium as

\[ p \partial \sum_{i=1}^{n} M_i u_i / \partial t = \sum_{i=1}^{n} M_i \partial u_i / \partial x_i = \sum_{i=1}^{n} M_i \partial u_i / \partial x_i + P_0 \]

(1.5)

and those for the overall total energy as

\[ \sum_{i=1}^{n} \left( M_i \left( \varepsilon_i + \frac{u_i^2}{2} \right) + a_i u_i p \right) = \sum_{i=1}^{n} \left( M_i \left( \varepsilon_{i0} + \frac{u_{i0}^2}{2} \right) + a_i u_{i0} P_0 \right) \]

(1.6)

Here \( u_i = \varepsilon_i - U \) is the mass speed of the particles of phase \( i \) relative to the shock-wave front; subscript zero denotes quantities characterizing the state of the medium in front of the shock wave. The equations for momentum and energy balance for phase \( i \) become as follows in terms of the new variable:

\[ \frac{d}{dz} \left( M_i w_i + a_i p \right) - p \frac{\partial}{\partial z} = R_i \]

(1.7)

\[ \frac{d}{dz} \left( M_i \left( \varepsilon_i + \frac{w_i^2}{2} \right) + a_i p w_i \right) - U p \frac{\partial}{\partial z} + y_i + q_i = 0 \]

(1.8)

We multiply (1.7) by \( U \) and subtract from (1.8) to get

\[ \frac{d}{dz} \left( M_i \left( \varepsilon_i + \frac{w_i^2}{2} \right) + a_i p w_i \right) - a_i p (w_i - U) - M_i w_i U \right) = - q_i - y_i - R_i U \]

We integrate this equation over the range \( z_0 - h, z_0 + h \), which includes the shock-wave front, and then pass to the limit \( h \to 0 \), which gives us the following relationship on the assumption that the range \( 2h \) allows unlimited increase only in the derivatives, while the variables themselves \( w_i, \varepsilon_i, p \), and the functions \( q_i, y_i, \) and \( R_i \) of these vary stepwise, but with restricted changes:

\[ M_i (\varepsilon_i + u_i^2 / 2) + a_i p = M_i (\varepsilon_{i0} + u_{i0}^2 / 2) + a_i u_{i0} P_0 \]

(1.9)

2. Mechanical Equilibrium Approximation. We write (1.4)-(1.6) and (1.9) for the case where mechanical equilibrium between the phases is attained behind the shock-wave front, i.e., we assume that immediately behind the step we have

\[ u_1 = u_2 = \ldots u_n = u \]

This relationship closes system (1.4)-(1.6) and (1.9).