Minimum weight shape and size optimization of truss structures made of uncertain materials

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Abstract Truss structures are optimized with respect to minimum weight with constraints on the value of some displacement and on the member stresses. The truss is considered made of an uncertain material, i.e. the value of the material constants are not known in a deterministic way, and each member may then exhibit a different value of stiffness, within a limited range of variation. The optimization must be done so that optimal solutions remain feasible for each value that the material constants may take for the considered uncertainty. In the present work a nonprobabilistic approach to uncertainty is used, and a variation of the material moduli with a, probabilistically speaking, uniform distribution over a convex and linearly bounded domain is considered. The two-step method is used to include the uncertainty within the optimization, where a diagonal quadratic approximation is used for the objective function and the constraints. Solutions for some of the most classical truss examples are found and compared with those obtained using nominal values of material constants.

1 Introduction

The main goal of structural optimization is to find out how to best exploit the material and the geometry of a structure so as to maximize or minimize a cost function within a feasible domain defined by some constraints. The cost function is in general assumed as the weight or the volume of the structure, or some structural response, as for instance displacements or stresses, or else the eigenvalues which determine the dynamic behaviour or the load that may lead to unstable behaviour. Of all these options the weight is in general chosen as the objective of the optimization, while some structural response control the search under the form of problem constraints. In these cases the optimization would give the lightest or the cheapest structure that is able to perform a given structural task.

When uncertainty is considered the optimization problem will modify so as to be able to evaluate the feasibility of the solution, i.e. to verify the constraints, within the assumed domain of uncertainty. This is particularly important in optimization because structure resources are exploited at their maximum level and in general the optimal design lies on the boundary of the unsafe zone or is at the minimum acceptable distance from it, and variation of the uncertain parameters could easily make the design unsafe. If the uncertainty is given in a probabilistic fashion, the problem would be to find a solution with a probability of constraint violation below a certain value. This approach leads in general to fine results, but requires a certain level of information to define the probability density functions of the parameters. Ben-Haim and Elishakoff (1990) showed that an imprecise definition of the PDF may lead to unacceptably large errors in the results, and suggested alternative methods when limited information is available. Furthermore having to deal with PDF rather than constants involves great computational effort in optimization where hundreds of different designs are generated during the search and verified against constraint violation.

When the knowledge of the uncertainty is limited, alternative approaches may be taken as, for instance, convex modelling and fuzzy sets. Convex modelling was developed by Ben-Haim and Elishakoff (1989, 1990) and proposed as a useful and practical way to evaluate the least favourable response of a structure subject to uncertainty. This consist in defining intervals of variation for the uncertain data and search within this domain for the least favourable combination for the considered structural response. This search was termed antioptimization by Elishakoff (1990). In this way one obtains the variation of the response for the assumed level of uncertainty, represented by the size of the uncertain domain around the nominal value of the parameters. Ben-Haim and Elishakoff (1989) used antioptimization to evaluate the nonlinear buckling of a geometrically imperfect thin cylindrical shell, Elishakoff et al. (1994) compared convex modelling and the stochastic approach for the evaluation of buckling load of a column with initial geometric imperfection. The two approaches gave comparable results when the deviation from nominal geometry was not small while the convex method was too conservative for small deviations. Antioptimization was also used to find the condition maximizing the difference between alternative structural models. Gangadharan et al. (1991) compared two different models of a car welded joint, while van Wamelen et al. (1993) compared different failure criteria for composites. Lee et al. (1994) used antioptimization to detect delamination in a composite beam.

The use of convex modelling to optimize uncertain structure was first considered by Adali (1992). Adali et al. (1994) considered the optimal design of symmetric angle-ply laminates for maximum buckling load with scatter in material properties and Adali et al. (1995) found minimum weight design of symmetric angle-ply laminates under uncertain loads. In both works the worst condition for the laminate was derived analytically by imposing optimality conditions. This was made possible by the linearity of the antioptimization
objective function. Elishakoff et al. (1994) considered the optimization of uncertain truss structures. The authors showed how with general constraints the search for the worst condition, embedded in the optimization, creates a nested optimization problem, which may be computationally expensive to solve. Lombardi et al. (1995) separated the search for the worst conditions from the main optimization so that by solving iteratively optimization and antioptimization convergence can be reached in a small number of cycles. The method was tested on the optimization of beams, plates, and composite structures.

In the present work an optimal solution for the well studied problem of truss shape and size optimization will be searched when a certain level of uncertainty in the material constants is considered. Optimal truss geometry was considered by Dobbs and Pelton (1969) and by Vanderplaats and Moses (1972) who searched for minimum weight design with stress and local buckling constraints. Svanberg (1981) considered the shape and size optimization for minimum weight, with displacements, stresses and local buckling constraints, using analytical expression for the first- and second-order sensitivities. Galante (1996) used a genetic algorithm to optimize the shape of real-world trusses.

Optimization of truss structures with fixed geometry subject to uncertain loads was considered by Elishakoff et al. (1994) using nonprobabilistic description of uncertainty. Feasibility of designs over the uncertain domain was done within the optimization, and this limited the application to small trusses, or to a limited number of constraints. Lombardi (1998) solved the same problem using the two-step method proposed by Lombardi et al. (1995), showing good agreement of results and demonstrating a great potential to reduce the computational effort.

In the present study previous work on uncertain truss optimization is extended to consider size and geometry optimization with stress, and displacement constraints. Uncertainty will be limited to Young’s modulus and will be described using the convex set approach. Each bar in the truss may exhibit a different value of the modulus $E_i$ which is allowed to deviate from the nominal value $E_0$ so that $|E_i - E_0| \leq \delta$, where $\delta$ is the considered level of uncertainty. The uncertainty vectors $E_j = \{E_i\}_j$, producing the worst occurrence of each constraint, are evaluated outside the optimization with a standard nonlinear line search algorithm. Optimization is carried out using an SQP algorithm. In each subproblem the constraints are approximated with a second-order Taylor expansion where the diagonal quadratic approximation suggested by Zhang and Fleury (1985) is used to simplify the problem. First- and second-order sensitivities are analytically evaluated as suggested by Svanberg (1981). Numerical results are obtained for some of the most common truss examples, and compared with optimal trusses obtained using nominal material and with those obtained using a uniformly reduced Young’s modulus as if a safety factor were applied to material properties.

2 Problem description

Consider a general three-dimensional truss with $N_{EL}$ bar elements connecting $N_{NOD}$ nodes. The truss is subject to static deterministic loads acting on some of the unconstrained nodes of the structure and to displacement and stress constraints. Least weight designs satisfying all the requirements are sought. This constrained minimization problem can be stated as

$$\min_{A_i, x_t} W(A, x),$$

subject to

$$\left| u_j(A, x, E) \right| \leq u_{j,max}, \quad j = 1, \ldots, N_u,$$

$$|\sigma_m(A, x, E)| \leq \sigma_{m,adm}, \quad m = 1, \ldots, N_{EL},$$

$$A_i \in [A_i^{\min}, A_i^{\max}], \quad i = 1, \ldots, N_A,$$

$$x_{\ell} \in [x_{\ell}^{\min}, x_{\ell}^{\max}], \quad \ell = 1, \ldots, N_c,$$  \hspace{1cm} (1)

where $A$ is the vector of the element areas, $x$ is the vector of coordinates, $E$ is the Young’s moduli vector, $u_j$ and $\sigma_m$ are the current values of the generic constrained nodal displacement and element stress, $u_{j,max}$ and $\sigma_{m,adm}$ are the displacement and stress allowable bounds. The variable areas and nodal coordinates are allowed to vary between given bounds indicated by the superscript min/max; $N_u$ is the number of constrained nodal displacements and $N_A$ and $N_c$ are the number of variable cross-sectional areas and nodal coordinates; $N_A$ is upper-bounded by $N_{EL}$ while $N_c$ is limited by the number of degrees of freedom of the truss.

Displacements and stresses depend on the optimization variables as well as on the material properties, which may not be safely considered as deterministic. As a matter of fact, in designing structures one must always consider safety factors that artificially weaken the structure and increase the loads, to reduce the probability that the combined effects of material and load deviate from nominal values and produce a violation of structural constraints. It has been shown, however, that the worst effects are not produced by the lowest value of the material constants or by the highest values of the loads (Lombardi et al. 1995; Elishakoff et al. 1994). A better approach is to consider the uncertain parameters as variable in a predefined domain. When using the convex set approach this domain is in general an hypercube or an n-dimensional ellipse. The ellipse allows the analytical description of the domain boundary making it easier to analytically describe the worst value of the uncertainty, while the hypercube is easier to manage since the constraints on each individual variable are linear. In the present work the uncertainty domain $\Omega_E$ will be of the latter kind, i.e.

$$\Omega_E = \left\{ E \in R^{N_{EL}} : E_i \in [E_i^0 - \delta_i, E_i^0 + \delta_i] \right\},$$  \hspace{1cm} (2)

where the $\delta_i$’s define the level of uncertainty, which can be different for each variable, although in the following a unique value of $\delta$ will be used.

In (1) it is required that the constraints be verified for all values $E \in \Omega_E$, i.e.

$$u_j(A, x, E) \leq u_{j,max} \quad \forall E \in \Omega_E,$$

$$|\sigma_m(A, x, E)| \leq \sigma_{m,adm} \quad \forall E \in \Omega_E.$$  \hspace{1cm} (3)