HALFBAND FILTERS AND HILBERT TRANSFORMERS*

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Abstract. This paper presents in summarizing form a description of halfband filters and the related symmetrical Hilbert transformers. It starts with the two complementary relations by which halfband filters are defined and the consequences for their impulse responses. The idealized versions of the frequency responses of halfband lowpasses and Hilbert transformers are introduced, and the related tolerance schemes that realized systems must satisfy are described. Using their frequency responses, the transformation of one filter type into the other is presented in general form. The design of finite impulse response (FIR)-halfband filters and their relation to corresponding Hilbert transformers are recalled, using maximally flat and Chebyshev approximations as examples. It is shown that the relation between both types of systems can be used for the infinite impulse response (IIR) case as well. The design of IIR-halfband filters is presented for systems with approximately linear phase and for those with minimum phase again for maximally flat and Chebyshev approximations. The design methods are partly new. The general procedure for the transformation into Hilbert transformers yields noncausal solutions, one of which is already known from the literature. By modifying this operation, phase-splitting systems are obtained, one of them related to corresponding continuous ones, discussed in papers published around 1950. Another system with approximately linear phase corresponds to a paper presented in 1987. Finally, the coupled form of these phase splitting allpasses is found to be a Hilbert transformer with precise phase difference, but with deviations of the magnitudes of the frequency responses.

1. Introduction

Halfband filters and Hilbert transformers are very interesting devices, first for their applications: halfband filters for decimation, interpolation, and multirate systems in general; Hilbert transformers for single-sideband transmission systems. But they are also of interest from a theoretical point of view. Halfband filters can be defined using two different complementary relations between the lowpass- and the

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corresponding highpass-subsystem, yielding special properties of their transfer functions. The frequency responses of Hilbert transformers have, according to an appropriate design and the corresponding implementation, either the desired ideal phase or ideal magnitude. Furthermore, FIR as well as IIR versions are possible for halfband filters and Hilbert transformers, and — most interesting — transformations are available for the mapping of one system into the other.

We give a short historical background. The interest in the generation of two time functions being mutually Hilbert transformed began around 1950 in the context of the design of single-sideband transmission systems. A large number of papers dealt with the design of continuous allpasses, the phase responses of which differ by approximately $\pi/2$ [9], [24], [30], [21], [44], [4]. The digital version of the systems, found by bilinear transformation, has been presented in [16]. Transversal filters, an implementation of nonrecursive systems in the continuous domain, were used for Hilbert transformation in the mid-1960s (e.g., [33], [32]). The first design of digital FIR-Hilbert transformers with Chebyshev approximation of the desired frequency response was published in 1969 [18]. This paper also described the transformation of these systems into the corresponding halfband filters, a basic result, presented again in [20] and in a larger context in [26].

FIR-halfband filters found their first application in the solution of interpolation and decimation problems in 1974 [5]. They can be designed by specializing the well-known methods for FIR filters of different types. An interesting way based on upsampling the impulse response of a fullband filter was described in [42]. The filter applications have been the subject of several books (e.g., [7], [41]).

IIR-halfband filters are seen to be in relation to the implementation of recursive systems as coupled allpasses, first introduced as wave digital filters [12], [13], [46], and later on used without relation to continuous systems (e.g., [31], [28]). The first IIR-halfband filter was described in [45]; different methods for the design and application of the filters in sampling rate alteration were discussed in [2], [3]. In those papers the case of halfband filters with approximately linear phase was treated as well. The transformation of an IIR-halfband filter into a phase splitter was presented in [1]; the design of an IIR-Hilbert transformer, to be implemented as a wave digital system, in [23]. Finally the design of phase splitters with approximately linear phase was described in [34].

This paper presents in summarizing form the properties and the design of halfband filters and Hilbert transformers. Their relations are treated in a general form, to be applied to the FIR as well as to the IIR case. First the two complementary relations for halfband filters are presented, yielding characteristic properties of their impulse responses. The definitions of ideal halfband filters and Hilbert transformers are given, based on their description in the frequency domain. These ideal properties are to be approximated by those of realizable systems. Thus they can be used for a specific application if they satisfy certain conditions, expressed by tolerance schemes for the components of their frequency responses. The transformations of one type of system into the other are described in the frequency domain.