LINEAR DYNAMICAL SYSTEMS WITH PARTIAL DERIVATIVES

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Systems with multidimensional time over Lie-algebras are considered. It is proved that a transfer function determines a minimal system up to similarity.

1. INTRODUCTION

In this paper we present a generalization of ordinary system theory to multidimensional systems. The basic notions such as transfer functions, coupling of systems, inverse systems, observability and reachability are extended to a wide class of systems with partial derivatives. In particular, we consider the following class of systems,

\[ \frac{\partial x}{\partial t} + \sum_{i=1}^{n-1} A_i \frac{\partial x}{\partial s_i} = A_0 x + BU \]

\[ y = C_0 x - \sum_{i=1}^{n-1} C_i \frac{\partial x}{\partial s_i} + U \]

For ordinary finite dimensional systems there is a one to one correspondence between minimal systems (up to similarity) and rational matrix-functions: their transfer functions. In our case even minimal systems are not generally determined by their transfer functions. This is mainly because operators \( \{A_i\}_{i=0}^{n-1} \) do not commute. This makes it natural to consider systems such that their main operators \( \{A_i\}_{i=0}^{n-1} \) form a Lie-algebra.

It is proved in the second section that two minimal systems with partial derivatives over Lie-algebras with the same structure constants have the same transfer function only if they are similar. Then the question was whether two nonsimilar
minimal systems over Lie-algebras with different structure constants could have the same transfer function. An example of such systems was easily constructed, and it indicated that further assumptions about systems should be made.

In the third section we consider systems such that their main operators \( \{A_i\}^{n-1}_{i=0} \) form Lie-algebra as well as the main operators of the inverse system \( \{A_i - BC_i\}^{n-1}_{i=0} \) with the same structure constants. We assume also that both the input and the output operators are one sided invertible. It is proved that in this case minimal systems with the same transfer function should be similar.

This paper is based on operator theory. We consider a transfer function of a system with partial derivatives as a generalization of a characteristic function of operators colligation introduced in [2] by M. Livšic et al. In operators colligation operators act in a Hilbert space and an imaginary part of operator \( A_i \) is considered instead of an arbitrary \( BC_i \). Essential work on this topic has been done by H. Gauchman [3], N. Kravitsky [4] and L. Waxman [5]. In fact the Similarity Theorem I of the second section is a generalization of [5] in finite dimensional case. As a corollary of Similarity Theorem II we were able to obtain a stronger result than that in [5] under the assumption that \( \varphi \) is right invertible.

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2. A GENERALIZATION OF LINEAR DYNAMICAL SYSTEMS

In this section we attempt to generalize the notion of dynamical system with as few restrictions as possible.

2.1. Definitions. We consider the following class of linear dynamical systems,

\[
\frac{\partial x}{\partial t} + \sum_{i=1}^{n-1} A_i \frac{\partial x}{\partial s_i} = A_0 x + Bu
\]

\[
y = C_0 x - \sum_{i=1}^{n-1} C_i \frac{\partial x}{\partial s_i} + u
\]