It is shown that fine-scale turbulent motions of a viscoelastic fluid damp out as in a viscous fluid with some effective viscosity dependent on the scale of the motion. The elasticity of deformation results in a diminution in the dissipativity of the turbulence, and hence, to an elongation of the high-frequency tail of the spectrum for a given energy influx.

1. The distinguishing peculiarity of viscoelastic fluid flows (the elasticity is essential for flows of solutions and polymer melts) is stress relaxation and the aftereffect of the strain rates, i.e., in some "memory" of the medium. At least two* constants are needed in their model description: the coefficient of viscosity \( \nu \) and the local relaxation time \( \theta \). The corresponding motions of scale \( l \) with characteristic velocity \( v_l \) will depend on two dimensionless complexes: the Reynolds number \( R_l = v_l \theta / \nu \) and the so-called Weissenberg number \( W_l = \theta v_l / l \). If the number \( W_l \) is small, then the motions will not differ from viscous fluid motions.

In the turbulent flow of a fluid with slight elasticity, when \( W = \theta V / L \ll 1 \) for the main flow, the large-scale motions do not experience the influence of elasticity. Nevertheless, the fluid elasticity can strongly influence the fine-scale motions for which the magnitude of the parameter \( W_l \) will reach one. Indeed, by using the formula of the theory of viscous fluid turbulence \([1, 2]\) \( l / L \sim (R_l / R)^{1/4} \), where \( R = V L / \nu \), we obtain \( W_l \sim W (R/R_l)^{1/4} \). It is seen from these relations that as the scale diminishes the Reynolds number \( R_l \) diminishes and the Weissenberg number \( W_l \) increases. Since \( R_l \sim 1 \) for the finest-scale turbulent motions, then, the influence of elasticity on turbulence can be significant when the number \( WR^{1/2} \) reaches one in order of magnitude, i.e., in sufficiently developed turbulence.

Important hydrodynamic singularities are associated with the nonlinear nature of the relaxation and aftereffect of a moving viscoelastic medium. The stress state turns out to be more complex in its flow than in the ordinary viscous fluid, and unequal normal stresses ("the effect of normal stresses") appear even in a simple shear flow. Diverse viscosities can be introduced by defining them as the ratio between diverse stresses and the strain rates, where such viscosities will depend on the strain rate ("non-Newtonian viscosity").†

In general, higher strain rates (\( \sim v_l / l \)) correspond to lesser motions in developed turbulence so that the viscosity of the fine-scale motions, for which \( 1 / \theta \sim v_l / l \), will vary most strongly in the case of monotonically varying non-Newtonian viscosity; i.e., the Weissenberg parameter is on the order of one. For such motions it is possible to speak of a certain effective viscosity dependent on the scale of the motion. As regards the additional normal stresses which differ from the hydrostatic pressure and ordinary Reynolds stresses, they then increase as the fluid elasticity grows and also exert influence primarily on the fine-scale motions.

*Henceforth, the fluids will be considered incompressible, and the units of measurement are selected so that the density equals one.
†The non-Newtonian viscosity and the effect of normal stresses are characteristic of not only viscoelastic fluids, but also for such media without relaxation as a nonlinearly viscous fluid with nonlinear coupling between the stresses and strain rates \([2]\).
By extending the concepts of the theory of locally isotropic viscous fluid turbulence \[2\] to a viscoelastic fluid, we can write for the spectra of the kinetic and elastic energies of the motions of an equilibrium interval of wave number \(k\)

\[
E_k(k) = \langle e \rangle^{\nu_{1}}|k^{1/2}f_1(k\eta, kl)\rangle
\]

\[
E_k(k) = \langle e \rangle^{\nu_{2}}|k^{1/2}f_2(k\eta, kl)\rangle
\]

Here, \(f_1\) and \(f_2\) are dimensionless universal functions of the two variables \(k\eta\) and \(kl\eta\). \(\langle e \rangle\) is the total energy dissipation into heat, \(\eta = \nu^{3/4}(\xi)^{-1/4}\) is the characteristic length of the dissipative turbulent motions, and finally, \(l_\eta = (\nu\eta)^{1/2}\) characterizes the dimension of the zone of influence of the relaxation process (the elastic response of the medium).

If \(l_\eta \lesssim \eta\) in fluids with low elasticity, the local and viscous relaxations for large-scale equilibrium motions \((k\eta \ll 1)\) terminate in a time which is small as compared with the motion lifetime, and the kinetic energy spectrum in the inertial interval of wave numbers will be independent of the elasticity of the medium. The fluid elasticity exerts substantial influence primarily on the fine-scale motions of the dissipative interval for which \(kl_\eta \gg 1\).

2. Fine-Scale Turbulent Motions \((k\eta \gg 1)\). Fine-scale dissipative motions are in an equilibrium state subjected to the distorting influence of the larger motions and the energy dissipation into heat \([2, 3]\). Hence, the problem of the behavior of such motions can be reduced to the problem of the behavior of small perturbations in a velocity field \(a_{ij}\). The tensor \(a_{ij}\) is a constant symmetric strain rate tensor in the coordinate system rotating together with the fluid particle, for whose principal values

\[
a_1 + a_2 + a_3 = 0, \quad 0 < a_1 > a_2 > a_3 < 0
\]

Small velocity pulsations \(v_i(x, t)\) in a system of coordinate axes coupled to the principal axes of the tensor \(a_{ij}\) satisfy the equations

\[
\frac{\partial v_i}{\partial x_i} = 0
\]

\[
\left(\frac{\partial}{\partial t} + a_{ix} \frac{\partial}{\partial x_i}\right) v_i + a_{ij} v_j + \frac{\partial p}{\partial x_i} = \frac{\partial \sigma_{ij}}{\partial x_j}
\]

Here, \(p\) is the pressure, \(\sigma_{ij}\) is the stress tensor; the summation is understood here and henceforth to be over the repeated subscripts denoted by the Greek letters.

The equation for the vorticity

\[
\omega = \delta_{ij} \frac{\partial v_j}{\partial x_i}
\]

does not contain the pressure \(p\). (\(\delta_{ij}\) is a unit completely antisymmetric tensor.)

\[
\left(\frac{\partial}{\partial t} + a_{ix} \frac{\partial}{\partial x_i}\right) \omega_i - a_{ij} \omega_j = \delta_{ij} \frac{\partial \sigma_{ij}}{\partial x_i \partial x_j}
\]

In order to close the system of equations under consideration, it is necessary to find the relationship between the stresses and the velocities for fine-scaled motions by using the governing equation of the medium. The governing equation for a large class of viscoelastic fluids can be written as an operator relationship connecting the stress tensor, the strain rate tensor, and the Jaumann differential operator \(\Delta/\Delta t\), which acts on the tensor \(\tau_{ij}\) as follows:

\[
\left(\frac{\Delta \tau_{ij}}{\Delta t}\right) = \frac{\partial \tau_{ij}}{\partial t} + \nu \frac{\partial \tau_{ij}}{\partial x_i} - \frac{1}{2} \omega_a \left(\delta_{ia} \tau_{ji} + \delta_{aj} \tau_{ji}\right)
\]

* The theory of locally isotropic turbulence can be refined if the fluctuation in dissipation is taken into account \([2]\).