NONUNIFORM ROTATIONAL FLOW OF A COMPRESSIBLE GAS THROUGH A LATTICE OF PLATES

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UDC 533.6.011.34

The problem of flow through a lattice of plates that moves in a plane-parallel subsonic stream of ideal gas with a small velocity inhomogeneity, having a nonpotential character, is solved. It is shown that as this takes place, monochromatic pressure waves are generated, whose frequencies are multiples of the frequency of passage of the plates of the lattice. Analytic expressions for the intensity of these waves and also for the magnitude of the radiated acoustic power and its spectral composition are obtained.

In [1] it has been shown theoretically that when an annular lattice of small depth is rotating in a cylindrical channel, a pressure inhomogeneity associated with the flow through it causes the appearance of pressure oscillations in the gas with discrete frequencies that are multiples of the frequency of passage of the blades of the lattice (product of the rotation frequency with the number of blades $N_z$). However, the energy of these oscillations propagates along the axis of the channel and radiates from it only for supersonic rotation velocities of the lattice, because at subsonic rotation speeds a potential type of pressure inhomogeneity rapidly attenuates with distance from the lattice and at the entrance section of the channel is practically nonexistent. At the same time experiments have shown that in the case of rotation of an isolated blade impeller at subsonic speeds considerable acoustic energy is radiated from a long cylindrical intake channel with the generation of discrete noise with intensity up to 120-130 dB.

To explain this discrepancy a hypothesis is proposed, according to which the generation of pressure waves and the radiation of acoustic energy occurs as the result of interaction of the lattice with a small peripheral inhomogeneity in the velocity of the oncoming stream, which is always the case in actual streams. The flow of a nonhomogeneous stream of incompressible fluid through a lattice of profiles has been studied in [2-5].

Below we obtain the solution to the problem of nonuniform rotational flow of a compressible gas through a lattice of plates. This makes it possible to calculate the acoustic wave phenomena that arise during the process. In solving the problem, we have employed certain results of [6].

1. We consider a steady flow in the $zy$ plane of an ideal compressible gas, whose streamlines are parallel to the $z$ axis and whose velocity is constant along each streamline, but changes from streamline to streamline according to a certain law.

Such a flow can be treated as a homogeneous one, moving with a velocity $U_0$ that is constant throughout the entire plane, upon which is superposed a free vorticity, which at each point causes the addition of a certain perturbation $u_b$ to the basic velocity, so that the total velocity is

$$U = U_0 + u_b$$

In what follows we shall assume that $u_b \ll U_0$, so that we can confine ourselves to a linear approximation. It is easily shown that in such a flow the pressure $p$ is constant everywhere, the density $\rho$ is constant along each streamline, and the law of variation of the density across the streamlines may be arbitrary, not depending on the way the velocity varies.

Suppose that a lattice of plates is moving with the velocity \( u_k \) in the positive direction of the \( y \) axis (Fig. 1, the stagger angle \( \theta \) of the lattice being such that, so far as the relative motion of the basic flow is concerned, the flow about the plates has zero angle of attack). Required is the determination of the field of secondary perturbations, introduced into the flow by the moving lattice.

The lattice under consideration represents the development on a plane of the cylindrical cross section of a three-dimensional blade impeller, consisting of \( z_p \) blades with spacing \( t_0 \) (in the cross section in question). In view of this, the velocity perturbation is a periodic function of the \( y \) coordinate with a period equal to the circumference of the cylindrical cross section. This function can be developed in a Fourier series

\[
 u_b(y) = \sum_{m=1}^{\infty} A_m \sin \left( \frac{2\pi m}{2\pi t_0} y + \varphi_m \right)
\]

In the linear formulation it suffices to obtain the solution of the problem on the interaction of the lattice with any one of the harmonics.

In complex notation the \( m \)-th harmonic of the velocity inhomogeneity has the form

\[
 u_b(z, y) = Ae^{i\omega t}, \quad v_b(z, y) = 0
\]

where \( u \) and \( v \) are the components of the velocity perturbation along the \( z \) and \( y \) axes, respectively, and

\[
 \mu = 2\pi m (z, y, t)^{-1}
\]

We transfer to a coordinate system \( z', y', \) attached to the moving lattice, the \( z' \) axis being directed counter to the relative velocity \( U_0' \) of the basic flow. The transformation formulas have the following form:

\[
 z = z' \cos \theta - y' \sin \theta, \quad y = z' \sin \theta + y' \cos \theta + u_0 t
\]

where \( t \) is the time. The components of the velocity inhomogeneity are then

\[
 u_b(z', y', t) = A \exp \left[ -i \left( \frac{k}{M} z' + \frac{k}{M} c_{d*} y' + \omega t \right) \right] \cos \theta
\]

\[
 v_b(z', y', t) = -A \exp \left[ -i \left( \frac{k}{M} z' + \frac{k}{M} c_{d*} y' + \omega t \right) \right] \sin \theta
\]

where \( \omega = \mu u_k \), \( k = \omega / a \), \( a \) is the sound speed in the unperturbed flow, \( M = U_0' / a \), and \( c_{d*} = U_0 / u_k = \text{ctg} \ \theta \).

Thus, in the system of coordinates attached to the lattice, every harmonic of the inhomogeneity represents a travelling vorticity wave, causing the appearance of oscillatory harmonics of velocity perturbation in the gas, whose directions of propagation do not coincide with the direction of the velocity of the unperturbed flow. The angular frequency of these oscillations is

\[
 \omega = \mu u_k = 2\pi m N
\]

A velocity perturbation generated by a vorticity wave has a component that is normal to the surface of the plates of the lattice. Therefore, on the plates the impenetrability condition must be fulfilled; that is, the normal velocity component must vanish. In the gas a supplementary perturbation appears, compensating the normal velocity component in the vorticity wave and causing the appearance of secondary pressure waves, which propagate from the lattice in all directions.

2. We consider first the interaction of the vorticity waves (1.2) with a lattice of semiinfinite plates

\[
 y' = (r + \frac{1}{2}) t_0 \cos \theta, \quad -\infty < z' \leq (r + \frac{1}{2}) t_0 \sin \theta \quad (r = 0, \pm 1, \ldots)
\]

which is obtained from the lattice shown in Fig. 1 if each plate is extended to infinity in the negative \( z' \) direction. We seek the parameters of the resulting flow in the following form: