APPONIMATE SOLUTION OF THE PROBLEM
OF THE BOTTOM ZONE IN A RAREIFIED GAS

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The problem of the steady-plane monatomic rarefied gas flow around a semiinfinite bar is considered (the plane stationary case of the problem about the bottom zone). The problem is solved numerically at the level of the Krook relaxation model [1, 2]. A dependence of the gas density, velocity, and temperature in the whole flow domain on the space coordinates is obtained for supersonic and subsonic gas streams flowing around a body by using calculations on an M-20 electronic calculator.

1. The Krook equation can be written as

\[ f = Vf. \]  

Here (in the stationary case) \( f = f(r, u) \) is the distribution function of gas molecules over the coordinates and velocities, and \( f_0(r, u) \) is the local Maxwell distribution function

\[ f_0 = n \left( \frac{m}{2\pi kT} \right)^{3/2} \exp \left[ -\frac{m}{2kT} (u - U)^2 \right] \]

\[ n = n(r) = \int \int \int f \, du, \quad U = U(r) = \frac{1}{n(r)} \int \int \int u \, du \]

\[ T = T(r) = \frac{m}{3nk} \int \int \int (u - U)^2 \, du \]

where \( k \) is the Boltzmann constant, \( m \) is the mass of one molecule, \( n(r) \) is the number of particles per unit volume, \( U(r) \) is the macroscopic velocity of gas motion, \( T(r) \) is the gas temperature at the point \( r \), \( u = (u_x, u_y, u_z) \), \( du = du_x du_y du_z \) is the velocity of molecule motion, \( V \) is the integral kinetic operator.

Let us start from the system of integral kinetic equations proposed in [3, 4]. Then (1.1) is written in expanded form as

\[ f(r, u) = \int \Phi_e(r - ru, u) \Pi_o(r, u, \tau) \, d\tau \quad (u \notin \Omega(r)) \]

\[ f(r, u) = \frac{1}{\Pi_o(r, u, \tau)} \Psi_e(r, u) \Pi_o(r, u, \tau) + \int \Phi_e(r - ru, u) \Pi_o(r, u, \tau) \, d\tau \quad (u \in \Omega(r)) \]

\[ \Pi_o(r, u, \tau) = \exp \left[ -\int Q_e(r - qu, u) \, dq \right] \]
\[ Q_s(r, u) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |u - u_i| \sigma(|u - u_i|) f_s(r, u_i) \, du_i \]

\[ \Theta_s(r, u) = f_s(r, u) Q_s(r, u) \quad \text{(1.3)} \]

\[ \Psi_s(r, u) = -\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (u_i) f_s(r, u_i) \Theta(u_i, u, 0) \, du_i \quad \text{(1.4)} \]

Here \( \Pi_0(r, u, \tau) \) is the probability of free molecule motion at the point \( r \) with velocity \( u \) in the time \( \tau \), \( \Phi_0(r, u) \) and \( \Psi_0(r, u) \) are the inner and boundary generation functions corresponding to the local Maxwell distribution function, \( \sigma \) is the collision cross section, \( \Theta(u_i, u, \theta) \) is the boundary shock transform, \( \theta = \theta(r_s) \) is the temperature of the body surface, \( r_s \) is the radius-vector of a point on the surface of the streamlined body, \( n \) is the outer normal to the surface, \( T_s \) is the time of free molecule motion from the body surface to the point \( r \), \( \Omega(r) \) is a cone in velocity space which contains all the velocities \( u \) with which the molecule can arrive from the body to the point \( r \) without collisions. The specific form of the functions \( \sigma, T, \) and \( \Theta \) for the models of mutual gas-molecule interaction and with the surface of the streamlined body most often used has been examined in [5]. Since \( f_0 \) is the local Maxwell distribution function, (1.3) is satisfied only because of the conservation laws independently of the elastic collision mechanism [5].

Let us note that an analogous equation can be considered in addition to (1.1) by selecting the second approximation of the distribution function obtained by the Chapman–Enskog method [6] corresponding to a viscous gas in the right side of (1.4) in place of \( f_0 \), which will correspond to taking more exact account of the boundary conditions.

Let us construct a system of equations in the functions \( n(r) \), \( U(r) \), and \( T(r) \). Taking account of (1.2), we obtain from (1.1)

\[ n(r) = \int_{-\infty}^{\infty} Vf_s(r, u) \, du, \quad U(r) = \frac{1}{n(r)} \int_{-\infty}^{\infty} u \cdot Vf_s(r, u) \, du \]

\[ T(r) = \frac{m}{3k(n(r))} \int_{-\infty}^{\infty} [u - U(r)]^2 Vf_s(r, u) \, du \quad \text{(1.5)} \]

2. Let us turn to a numerical solution of the system (1.5) for the case of steady rarefied gas flow around a semiinfinite bar. Let us consider the gas to consist of Maxwellian molecules. Then [5]

\[ \sigma(v) = \frac{\sigma_s}{v}, \quad \sigma_s = \text{const}; \quad Q_s(r, u) = \sigma_s n(r) \]

Let us introduce the coordinate system shown in Fig. 1. Let the stream at infinity be directed along the \( x \) axis.

Let us assume that the reflection of gas molecules from the surface of the streamlined body is diffuse and the body surface temperature is \( \theta(r_s) = T(r_s) \).

Then [5]

\[ \Theta(u_i, n, u, \theta) = \frac{2}{\pi} \left[ \frac{m}{2kT(r_s)} \right]^\frac{3}{2} u_i \exp \left[ -\frac{m}{2kT(r_s)} (u_i^2 + u_s^2 + u_s^2) \right] \]

\[ \Psi(r, u) = n(r) \left[ \frac{m}{2kT(r_s)} \right]^{\frac{3}{2}} u_i \exp \left[ -\frac{m}{2kT(r_s)} (u_i^2 + u_s^2 + u_s^2) \right] \times \]

\[ \times \left\{ \exp \left[ -\frac{m}{2kT(r_s)} U_i^2(r_s) \right] \right\} \]

\[ - \sqrt{\pi} U_s(r_s) \left( \frac{m}{2kT(r_s)} \right)^{\frac{3}{2}} \left[ 1 - \text{erf} \left( \frac{m}{2kT(r_s)} U_s(r_s) \right) \right] \]